INFLATION AND INDEXATION IN BRAZIL: THE INFLUENCE ON LIFE INSURANCE

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Inflation is a problem that has plagued economies worldwide, and its effects upon some basic institutions are of widespread concern. One of the economic institutions that is most susceptible to the effects of inflation is life insurance. In an effort to mitigate the impact of inflation upon the values specified in life insurance contracts, several countries have adopted index-linked life insurance policies. One country that has adopted such measures is Brazil. The investigation of this dissertation centers upon the effects of inflation on life insurance in Brazil, both before and after the advent of indexation.

The study begins with an inquiry into the nature and extent to which inflation affects the cost of achieving financial protection. It is found

that while the nominal cost of commodities, on average, accompanies the changes in the consumer price index, the nominal cost of life insurance will grow at a more rapid pace. In real terms, life insurance costs increase with inflation, as long as insurance regulators are slow to allow adjustments of policy terms to higher expected rates of inflation. The magnitude of this cost increase is shown to be extremely high in the case of Brazil, where policyowners do not participate in the insurance company profits. The real cost of life insurance protection through index-linked policies is also shown to rise with inflationary expectations.

After demonstrating the effect of inflation on life insurance costs, a theoretical model is developed to determine the influence of inflation upon consumer purchases of life insurance. Time-State Preference theory indicates that rational insurance purchases in real values will unambiguously be lower when inflation is anticipated, regardless of whether or not the policies feature indexing. The reason for the latter is that indexing in Brazil is not carried out on a continuous basis, but policy values are adjusted only annually, and on an ex post basis.

Finally, a multivariate time-series regression model is developed, and the Brazilian experience before and after indexing is tested to determine if inflation's impact on life insurance purchasing was negative, as hypothesized. The test results indicate that a negative relationship

between inflation (expected or realized) and net life insurance in force per capita prevailed in both the preindexing and postindexing periods of Brazil's history.

CHAPTER 1
INTRODUCTION

Background of the Study

Inflation is a problem that has plagued economies worldwide. Recently a growing number of countries have experienced double-digit rates of inflation which have affected basic institutions such as life insurance. The effects of inflation on life insurance contracts is especially pronounced due to two factors: these contracts are usually specified in fixed, nominal currency units, and most of them are designed to cover long periods of time. Because life insurance values are specified in fixed nominal currency units, they do not adjust to compensate for the value erosion produced by inflation, and because the contracts are generally long term, the accumulated erosive effects of inflation on the insurance values can be substantial. Thus, while life insurance products are designed to provide protection against the perils of longevity and premature death, inflation and a rising cost of living can undermine such protection.

Faced with the prospect of chronic inflation, the insurance industry has three strategic alternatives: (1) it may exert its influence in attempting to eliminate or reduce inflation; (2) it may accept higher

rates of inflation as inevitable, and adapt its products to such an environment; or (3) in the absence of success in either or both of the above, it can watch as its functions in society are usurped by other institutions, presumably government institutions.[1]

For a number of years insurance industry officials have engaged in sporadic attempts aimed at influencing governmental officials to take measures to arrest inflation. In the United States, for example, some insurance companies engaged in advertising campaigns to educate consumers as to the consequences of inflation on their financial security, thereby inducing the public to bring pressure to bear upon the government for fiscal and monetary restraint. Other companies directly lobbied the executive and legislative branches of government to exercise more restraint. However, most countries have lacked the will to take measures necessary to eliminate inflation, measures whose harsh consequences upon employment and economic activity can become politically unfavorable if not intolerable.

Having failed to make significant headway in controlling inflation, the insurance industry has turned toward developing insurance products

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[1]To fill the void, one of the governmental responses might be an expansion of social insurance programs such as the social security system operating in the United States. However, unlike life insurance, social insurance is often an essentially unfunded operation. Thus, to the extent that social insurance leads consumers to save less, adverse consequences can occur in the process of capital formation. For a discussion of this point and an estimation of the impact of social security on savings in the U.S., see Feldstein (1974).

better adapted to inflationary environments (Greene, 1974). One of the most promising alternatives is the index-linked life insurance contract.[2] In such contracts, the nominal values of the insurance premiums, cash values, and indemnification payments are linked to a price or cost of living index and adjusted periodically in accordance with changes that have transpired in the index. This approach has been taken in countries such as Brazil, Finland, and Israel.[3]

The feasibility of offering index-linked policies depends upon the existence of an asset in which the insurer can invest its proceeds which will consistently yield a rate of return commensurate with the rate of change in the index utilized. Without such an asset the insurer could face problems of capital inadequacy. Another consideration is that the ability of the insured to pay the fluctuating premiums must be at least loosely related to changes in the index used for adjusting the premiums. This constraint is less restrictive than the first due to the flexibility in the average consumer's budget, which is generally capable of absorbing temporary aberrations between the rates of growth in wealth and in premium expenditure obligations.

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[2]Other alternatives are discussed in the fourth section of Greene (1974).

[3]Finland and Israel have allowed partial inflation adjustments, whereas Brazil permits full inflation adjustments.

The Problem

Given an environment in which the existence of index-linked contracts is feasible, the insurer may ask if such contracts should be offered. If, on the other hand, conditions are such that the introduction of index-linked contracts is infeasible (e.g., no index-linked assets exist in which the insurer can invest insurance proceeds), the insurer may ask if its influence should be wielded in order to bring about the requisite conditions. The answers to these questions depend heavily upon the answers to three questions:

(1) Does inflation affect the cost of life insurance protection?

(2) Is the consumer sensitive to changes in life insurance cost wrought by inflation?

(3) Can index-linked life insurance contracts mitigate any adverse effects of inflation on life insurance costs and values?

If the answer to any of these questions is in the negative, further inquiry into the area of indexing and life insurance is unwarranted. If, on the other hand, the responses to these questions are in the affirmative, implications can then be drawn as to the advisability of developing and marketing index-linked life insurance policies.

Purpose of the Study

The three questions listed above constitute the subjects of investigation in this paper. The purpose of the study is to resolve these questions and the issues they entail through theoretical analysis, where

possible, and, in cases where a question is not amenable to such an analytical solution, to provide and utilize empirical analysis to help resolve it. The data for the empirical portions of the study will be taken from the Brazilian experience (for reasons to be discussed in the next section). While conclusions resulting from the theoretical analyses will have general applicability, those based on the empirical data will have inferences with respect to the Brazilian population under study. However, an effort will be made to determine the implications of the Brazilian experience with regard to the perspectives and potential problems of indexing life insurance policies in general.

Reasons for Studying the Brazilian Experience

This study will focus upon the life insurance industry of Brazil for two reasons: Brazil's long history of inflation and its extensive experience with indexation.

Almost all studies to date on the influence of inflation on life insurance demand have dealt primarily with the United States. The periods of study chosen have been ones of relatively low or unsustained moderate rates of inflation. There is possibly a threshold rate of inflation above which sales of long-term insurance are affected, i.e., where money illusion evaporates; if such is the case, empirical studies dealing exclusively with the United States may have overlooked or only marginally detected the disastrous effects that sustained inflation rates

above this threshold can have on insurance markets. Brazil's endemic double-digit inflation rates[4] make it an excellent candidate for studying the economic impact of inflation on insurance markets.

Since 1964, Brazil has undertaken systematic monetary corrections ("indexing") in an attempt to cope with rampant inflation and the distortions fostered by it.[5] In fact, indexing in Brazil has probably been more widespread than in any other country (Friedman, 1974). Thus, the Brazilian experience provides a rich base from which the effects that indexing can produce in actual practice can be investigated. It is anticipated that the Brazilian experience will provide some important insights into the benefits and problems of indexing for life insurance industries and for the consumers of life insurance in other countries suffering from inflation.

Justification of the Study

An inquiry into the influence of inflation and indexation on life insurance in general, and the Brazilian experience in particular, can be justified on three grounds. The issues involved are important: (1) to social welfare, (2) to capital formation, and (3) to the life insurance industry.

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[4] Brazil's prolonged experience with inflation is discussed by Buescu (1973).

[5] Fishlow (1974) describes in detail the system of indexing in Brazil.

The social significance of life insurance derives from its ability to protect covered members of society from fortuitous events that can produce disastrous aftermaths. Life insurance is used to provide protection against the financial consequences of premature death. It can also be used to accumulate a savings fund. In an environment of inflation, protection and savings available through life insurance are eroded. Consumers of insurance are thus exposed to financial risk[6] which insurance was designed to alleviate. If the consumers of insurance are unable to provide for their future, more pressure and responsibility may fall upon governments to do so. On the other hand, if indexing can restore life insurance as a viable instrument of saving and protection, people may wish to channel their resources in that direction.

Life insurance can also be important to capital formation, stemming largely from the fact that much of life insurance is sold in forms other than single year term insurance. These forms contain sizeable savings elements which provide funds to the insurance companies for financial investment. A large portion of these funds is generally channeled into the markets for long term investments to hedge against the associated insurance risk, whose nature is generally long term. Accordingly, life insurance has particular importance in the long term capital markets.

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[6]Financial risk refers to the possibility that a desired or required pattern of returns (across different possible future states of nature) will not be achieved.

A paralyzed life insurance industry can have crippling effects upon capital markets whereas a healthy life insurance industry can contribute significantly to their development. The extent to which inflation and indexation can paralyze and revitalize, respectively, a life insurance industry is therefore of great importance to capital formation.

Finally, the effects of inflation and indexation upon the demand for life insurance are important to the life insurance industry itself. The survival of the industry depends on its ability to attract consumers to purchase life insurance policies. If inflation diminishes consumer interest in the life insurance policies available through the industry, then the survival of the industry is in jeopardy.[7]

Review of the Literature

These vital roles of life insurance, and the adverse impact which inflation may have upon them, have received attention in the literature. While the influence of inflation upon the cost[8] of life insurance protection can be demonstrated analytically, no rigorous analytical treatments

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[7]The survival of the industry is not only important to the shareholders but also to those whose occupations are derived from the industry. The life insurance industry may employ large numbers of people, and through its investments provides employment for many more. As the industry loses vitality, it can no longer function effectively in this role.

[8]A thorough discussion of the concept of life insurance cost is presented in the next chapter.

of this problem were encountered in the published literature. There is, however, a study (Fitzhugh and Greeley, 1974) which does treat the problem through a retrospective example using actual policy data and historical inflation rates. While no real attempt was made at measuring the cost of life insurance under inflation, the article offers important insights into the options available through participating life insurance policies that can help offset some of the value erosion incurred by inflation.

The sensitivity of the consumer to changes in life insurance cost wrought by inflation can be examined through theoretical analysis, and resulting conclusions can then be tested empirically. To date there have been no rigorous theoretical treatments of the effect of inflation on consumer demand for life insurance. However, an attempt was made by Hofflander and Duvall (1967). They utilized budget constraints and indifference curves in a graphical analysis which they contend demonstrates that less life insurance (in real values) will be purchased if inflation is anticipated. Neumann (1968) correctly criticized their model and showed that the model can be used to demonstrate that purchases of life insurance protection could actually increase under inflationary expectations. While Neumann is correct in his criticism of the model, he is incorrect in using another version of the same model to substantiate his own theoretical analysis. The problem is that the model employed by the three authors is a timeless microeconomic model incapable

of properly taking into account the inflation and protection factors, which necessarily occur over time. Thus, none of the authors' propositions in this regard were derived with appropriate rigor.[9]

There have been a large number of empirical studies conducted to ascertain the determinants of demand for life insurance.[10] Some of the studies which used time-series regression analysis have included anticipated inflation as an explanatory variable. Of special interest are five articles dealing specifically with the effects of inflation on life insurance.

In an early study by David B. Houston (1960), the relationships between the price level and the pattern of savings through life insurance[11] in the United States were estimated through statistical techniques. The period of investigation was from 1919 through 1958. Houston concluded that there was no simple long term relationship between the cost of living and the extent of savings through life insurance, and that there was no indication that the life insurance industry has suffered as a result of the post-war inflation.

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[9]Their analyses are examined at length in Chapter 3.

[10]For a compilation of life insurance demand analyses conducted prior to 1970, see Lee and Whitaker (1970). A more recent review is in Headen and Lee (1974).

[11]The Houston study related savings through life insurance to the price level and not savings to "changes in the price level," as he misstated.

After indicating theoretically[12] that sales of both permanent and term insurance may decrease if there are anticipations of price level increases, Alfred E. Hofflander and Richard M. Duvall (1967) used two multiple regression models to test the relationship between price level changes and sales of life insurance in the United States. Their study covered a twenty year period beginning in 1945. The authors found that large increases in the cost of living have been accompanied by relatively smaller sales of term, as well as permanent life insurance.

A doctoral dissertation by Seev Neumann (1967) considers the impact of inflation on consumer savings through life insurance and arrives at a different conclusion. In his study of the period from 1946 until 1964, Neumann concludes that the data do not support the conclusion that consumer expectations of price changes[13] had any discernible effect on saving through life insurance in the United States economy. He allows, however, that "creeping" inflation might have a cumulative effect that takes time to influence the slow process of social learning.[14]

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[12]Misgivings with respect to the theoretical model used in the analysis have already been stated.

[13]In another article, Neumann (1969a) shows that theoretically it is anticipated inflation that has bearing on the problem and not the price level *per se*.

[14]Both the Hofflander/Duvall and the Neumann models use nominal, rather than real valued variables. In addition, the models are fraught with specification errors, to be discussed in detail later in this dissertation.

In a lengthy comment arising from the Neumann study, Peter Fortune (1972) produces a clearer and more precise theoretical discussion of the possible effects that inflation can have on savings through life insurance. After producing an alternative model,[15] Fortune uses quarterly data covering the period from 1953 until 1968 to test the propositions he states. He finds that the expected rate of inflation does have a negative impact upon optimum policy reserves per dollar of insurance through its effect on the relative real yields of financial and real assets, but that this effect is offset by other effects induced by inflation. Fortune concludes that inflation actually increases the flows of funds into the life insurance sector of the United States, but no evidence is presented with regard to the impact of inflation upon the flows into the life insurance sector relative to flows into other financial institutions. By such a relative measure the life insurance sector may be hurt by inflation.

A more recent study by Fortune (1973) develops a theory of optimal life insurance. Although his model does not accomodate the inclusion of inflation in a theoretical context, his empirical work includes an explanatory variable that is closely related to anticipated inflation,

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[15]The Fortune model uses financial variables, in contrast to the Hofflander/Duvall and Neumann models, which rely largely on socio-demographic variables.

and this variable was shown to be highly significant. More will be said of the Fortune statistical tests later in this paper.

An excellent review of these and other studies centering upon inflation and life insurance is given in a recently published booklet by Mark Greene (1974). Building on his article of twenty-one years earlier (1954), Greene discusses at length the many avenues through which inflation can and does affect the life insurance sector. Attention is first focused upon the impact of inflation upon life insurers, and then turns to inflation's effects upon the consumers of life insurance. In addition to the experience of the United States, Greene cites Canada and Colombia as indications that inflation has an adverse effect on life insurance demand.

The extent to which index-linked life insurance contracts can mitigate adverse effects of inflation on life insurance costs and values can be examined both theoretically and empirically. Economic literature is replete with articles on indexation.[16] However, to date neither theoretical nor empirical studies have been encountered dealing specifically with indexation of life insurance contracts.[17]

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[16] An OECD bibliography (1975) cites over 150 articles published until 1974 about indexation.

[17] Descriptive studies concerning the experience of index-linked life insurance contracts of Finland are given in Junnila (1965) and Ingman (1971).

Scope of the Study

A theory of inflation's impact upon life insurance demand has not been rigorously developed in economic and financial literature. In this study a model is developed which is capable of facilitating an analysis of the effect of anticipated inflation on rational consumer demand for life insurance protection. In addition, the theoretical remedial properties of indexation in the context of life insurance will be elucidated.

Statistical tests will be conducted to determine if the propositions derived from the theoretical discussions are substantiated in practice. The Brazilian case will be the basis for the tests, although tests already performed on the United States data base will also be examined for their relevance to the problems under investigation.

The period covered in the Brazilian data base is 1950 through 1976, a period of elevated and highly fluctuating rates of inflation. This period has been subdivided into the pre-1968 period (before indexing was implemented in life insurance contracts) and the post-1967 period (after indexing was adopted). This division is deemed to be appropriate for the purposes of the study, because the division isolates the issues under question for separate testing, while providing enough observations for statistical credibility.

Methodology and Format of the Study

This study is divided into four chapters. Following this introductory chapter, a capital budgeting approach for determining the cost of life insurance in an environment of inflation is developed in the second chapter. It is shown how different kinds of money illusion are implicit in some of the popular methods of evaluating the cost of life insurance, and a method is suggested which overcomes this problem. Analytical techniques are employed in demonstrating the effect of inflation on the cost of life insurance available through nonindexed and index-linked policy contracts. Finally, actual policy data obtained from a large Brazilian insurer are incorporated into the operational model to generate a sample array of life insurance cost data associated with various rates of anticipated inflation.

In Chapter 3, a theoretical model is developed for analyzing the effect of anticipated inflation on rational life insurance purchasing. The Expected Utility Hypothesis is used in a Time-State Preference framework to derive definitive propositions with regard to consumer demand for life insurance protection. The analysis is carried out for both nonindexed and index-linked policies. The outcome is a set of hypotheses concerning the relationship of anticipated inflation to consumer demand for life insurance.

After a discussion of the major determinants of consumer demand for life insurance, a time-series multiple regression model is developed in Chapter 4 for use in an empirical examination of the propositions derived in Chapter 3. The Brazilian experience, which serves as the data base for the statistical testing, is reviewed. Finally, the econometric model is used in testing the Brazilian data and the results of the tests are presented and interpreted. The chapter concludes with a summary of the major findings of this study, along with their policy implications, and suggests some areas for possible further research.

CHAPTER 2
MEASURING THE COST OF LIFE INSURANCE UNDER INFLATION

An Overview

In this chapter a methodology appropriate for measuring life insurance costs in an environment of inflation is developed. The model is then used to examine (1) the effect of inflation on life insurance costs, and (2) the extent to which indexation of policies can mitigate any adverse effects of inflation on life insurance costs and values.[1] To achieve unambiguous solutions to these problems will, at times, require some set of life insurance policy terms and provisions to be specified, as well as a knowledge concerning the influence of the institutional environment upon life insurance policy terms.[2] Since the focus of this study centers on the Brazilian experience, these problems will be viewed as they apply to the life insurance provisions and regulatory environment exhibited in Brazil. Accordingly, analytical models developed and utilized in this chapter will be adapted, where necessary, to the conditions prevailing in Brazil.

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[1]These problems are presented as two of the three research questions listed on page 4.

[2]In the absence of such information, a general analysis may result in ambiguous solutions.

A number of authors have devised methodologies by which the costs of life insurance may be computed.[3] Almost invariably, the methodologies have been developed and designed for use in comparing the costs of policies offered by differing companies. What is needed in this study is a method appropriate for measuring the changing cost of a given policy when subjected to an inflationary environment.

In designing a procedure appropriate for measuring inflation's impact on the cost of life insurance from the consumers' point of view, it is instructive to consider first a number of procedures that might be used which are inappropriate.[4] Such an approach is instructive in that the components of life insurance costing will be introduced in simpler forms, graduating in sophistication as a better understanding of the complexities of life insurance is gained. Moreover, this approach serves as a convenient vehicle for demonstrating the essential properties of an insurance costing procedure which adequately takes

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[3]Most of these are conveniently summarized in the nontechnical Report of the Joint Special Committee on Life Insurance Costs (1970). See also Belth (1966).

[4]The purpose here is not to disparage the methods that have already been devised. Most of the methods are appropriate in the applications for which they have been employed — comparing and ranking the costs of life insurance policies offered by different companies. In fact, there is some evidence (Kensicki, 1977) which suggests that all of the principal costing methods proposed in the literature yield similar rankings of policies. The purpose here is only to draw attention to the limitations of some of the methods if they are applied in estimating the costs of life insurance in an inflationary environment.

into account the effects of inflation, while alerting the reader to the shortcomings of (mis)applying some of the conventional costing methods presently in use.

In evaluating the effect of inflation upon the cost of life insurance, the consumer may be entrammeled by various degrees of money illusion.[5] The concerned consumer may estimate the (net) cost of life insurance in either nominal or real (or alternatively, present value) terms. Furthermore, he may link these costs to the nominal or real (or present) values of life insurance protection in force. Combining these alternatives leads to four general classifications of approaches which the consumer may take that are designated here as "money illusion," "partial money illusion," "policy illusion," and "no illusion." The degrees of consumer awareness in appraising the cost of life insurance are presented in matrix form in the table below.

TABLE 1

Consumer Approaches to Life Insurance Valuation

Consumer's Primary Focus:	Units of Nominal Protection	Units of Real Protection
Nominal (Net) Costs	Money Illusion	Partial Money Illusion
Real (Net) Costs	Policy Illusion	No Illusion

[5]By "money illusion" it is meant that the consumer, to some degree, bases his decisions on nominally valued economic data, rather than real-valued economic data.

In the four sections that follow, each of these approaches is examined in detail, and methodologies which have been suggested in the literature are reviewed as they relate to these approaches. After an appropriate method for costing has been developed, it will be applied to determine the impact of inflation on the cost of life insurance, and the extent to which indexation of policy values can alleviate any adverse impact.

Money Illusion

A consumer suffering from money illusion may realize that inflation is occurring, but fail to recognize the impact of inflation on the real costs and values of his life insurance policy.[6] "After all," he might remark, "the size of my premium has not gone up in spite of inflation." In fact, if insurance companies are able to lower the premium charge due to higher returns on investments from higher

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[6]The features most often included in life insurance policies are premiums, death benefits, cash values, dividends, and terminal dividends. The dividend features are available in "participating policies," but not in "nonparticipating policies."

All life insurance policies contain one or more of the three basic kinds of insurance: term, whole life, and endowment insurance. Term insurance features premiums and death benefits, with or without dividends. Whole life and endowment policies feature premiums, death benefits, and guaranteed cash values, with or without dividends. Term insurance policies offer financial protection against the peril of premature death; whole life and endowment policies offer protection against the peril of premature death and also offer cash savings, which can be used in providing protection against the peril of outliving one's earning capacity. For further details, see Pfeffer and Klock (1974).

interest rates, a consumer suffering from this degree of money illusion may even believe that the cost of life insurance is declining. His focus is on the nominal costs and nominal level of protection, and unless inflation affects these nominal values, the consumer does not recognize the impact of inflation on life insurance.

There are a number of specific costing procedures such a consumer might employ which may give rise to, or could serve to reinforce his illusion. One such method, commonly called the "Traditional Method," which has long been in use and which continues to be popular among consumers of life insurance,[7] shall be used here for illustrative purposes. Its procedure is to add together the insurance premiums for a number of years, usually twenty, and to subtract the sum of all policy dividends projected by the life insurance company for the period.[8] From the resulting figure is subtracted the cash value at the end of the period, and the final amount is then divided by twenty (or by the length of the period if other than twenty years), and by the number of thousands of the amount insured. The result, which may be

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[7]Indeed, in a survey by the Institute of Life Insurance (1974), consumers of insurance in the United States identified the Traditional Method as being the most "preferred" method.

[8]In actuality, the calculations use an illustrative dividend scale; the scale does not represent an estimate of what a company will pay, but rather, the current scale paid out on existing policies.

positive or negative, is the insurance cost per year per thousand currency units of life insurance in force. The calculation procedure may be represented by the following formula:

$$NC_k = \frac{\sum_{n=1}^{k} P_n - \sum_{n=1}^{k} D_n - CV_k}{k} , \qquad (1)$$

where

NC_k is the average net cost of insurance per year, per thousand currency units of life insurance in force for an insured who surrenders his policy at the end of year k;

P_n is the insurance premium payable at the beginning of year n, per thousand currency units of life insurance in force;

D_n is the insurance dividend receivable at the end of year n, per thousand currency units of life insurance in force;

CV_k is the guaranteed surrender cash value available to the insured in year k, per thousand currency units of life insurance in force; and

k is the year of policy surrender.

The Traditional Method could undergo refinements to reflect the probabilities of mortality and persistence, but the major drawback of the method is that it fails to give any recognition to the time when money is paid either by or to the policyholder. The focus is entirely on costs and coverage measured in nominal currency units. In none of the terms of equation (1) are inflation and interest even included as factors having bearing on the insurance values.

For years in the United States and elsewhere, short-sighted approaches to life insurance costing such as the Traditional Method were reinforced by some insurance agents who emphasized net cost calculations per thousand units of insurance in force to their clients. In countries with unstable currencies, such approaches can foster greatly distorted views of the true cost of life insurance, since the cost and benefit flows, which occur over long periods of time, may exhibit large ranges of differing real values. To avoid this distortion, some states in the United States now require use of an "Interest-Adjusted Method" to be used in calculating life insurance costs. More will be said about this in the section entitled "Policy Illusion."

Partial Money Illusion

While a consumer beset with money illusion would tend to believe that inflation has no effect or (if inflation leads insurers to lower the premium charges and/or increase the dividends and cash values) a reducing effect on the cost of life insurance, a consumer having partial money illusion would tend to think that inflation causes the cost of life insurance to increase. A consumer who reaches this stage is somewhat less naive in his thinking. He realizes that indemnification or cash surrender value, when actually received, will exhibit a value that has been eroded by the inflation prevailing in the period intervening the date when the policy is purchased and the date when it ends at death, surrender, or maturity.

This phenomenon has often led to criticism of life insurance products on the grounds that while premiums are paid in "good" money, benefits are received in "bad" money, i.e., money whose value has been eroded by inflation. The life insurance contracts in existence in Brazil before indexing was applied[9] were among those subject to this criticism, as shown by the quote that follows.

> After the Second World War, galloping inflation created in the public a lack of interest for insurance in general and especially for life insurance. While premiums were effectively paid in strong currency, indemnification, in the case of death, or cash savings, at the end of the policy period, which were fixed in nominal terms at the beginning of the contract, had a purchasing power infinitely less than the same quantities represented when the policy first came into force. Life insurance was thus abandoned, and savings were channeled into real estate, stocks and treasury bills.[10]

A more careful analysis, however, discloses that since life insurance is generally paid for year by year over a long period of time, the value of the premiums paid by the insured also declines over time with inflation; thus, premiums are not all paid in "good" currency. In addition, some of the benefits of life insurance (such as dividends, where available, and death protection) are received during the policy period rather than in a single lump sum settlement at the end, and consequently are not all received in "bad" money.

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[9] The application of indexation to insurance contracts was first authorized in Brazil by Decree-Law No. 73 of November 21, 1966.

[10] The translation of this quote into English, from Chacel et al. (1970, p. 255), is that of the author.

A consumer who fails to recognize this may be led to making a distorted appraisal of the effect of inflation on the cost of life insurance. Although no models in the published literature were found that reinforce this brand of (partial) money illusion, the notion that premiums are paid in "good" money while benefits are received in "bad" money appears to be fairly common with the "man in the street."[11]

Policy Illusion

A more subtle error is made when the consumer is myopic in his perspective of life insurance, wherein he considers the net cost (in real or present value terms) of a policy offering a given number of units of insurance in force. His focus is incorrectly on the vehicle (i.e., policy contract) rather than on the design (i.e., protection) of the life insurance purchase. Hence, it is denoted "Policy Illusion" in this discussion, and is actually a special subset of money illusion. Because of the subtlety of this type of money illusion relative to the first two types, more space will be devoted to its discussion.

In discussing this kind of money illusion, considerable care will be given to the elaboration of a capital budgeting procedure useful in measuring the impact of inflation on the costs and benefits associated

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[11]In interviews with a number of Brazilian citizens, a common reaction was that insurance was a poor investment in an inflationary context because premiums are paid in "good" money, benefits in "bad" money.

with life insurance. The effort will be well spent since the model will also be used in the subsequent section of this chapter, after undergoing slight modification to remove the final element of money illusion from the valuation procedure.

A Capital Budgeting Approach

Several authors have advocated a capital budgeting approach to the problem of consumer valuation of life insurance.[12] While such an approach in isolation cannot theoretically justify the purchase of life insurance (Friedman and Savage, 1948), it is useful in analyzing the costs and benefits in monetary terms, and is well adapted to cost comparisons. The principal advantage in using a capital budgeting approach is in its ability to account for the opportunity costs of money over time. Since a life insurance contract typically involves streams of payments and benefits over long periods of time, a capital budgeting approach is especially well suited for measuring the values of these flows.

Of the capital budgeting approaches available, the net present value (NPV) method is employed here because of its theoretical superiority (Hirshleifer, 1958) and its mathematical efficiency in providing annual cost comparisons (Kensicki, 1974). The net present value of an

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[12]The first study to treat the purchase of life insurance as a pure capital budgeting decision was Kensicki (1974). For a reader unfamiliar with capital budgeting techniques, see Brigham (1977, ch. 9,10).

insurance policy, per thousand units of insurance in force, can be estimated by the following formula:[13]

$$E\left[NPV_k\right] = -\sum_{n=1}^{k} \frac{P_n \frac{1}{(1-DR_{a-1})} \prod_{j=0}^{n-1} (1-DR_{a+j-1})}{\frac{1}{(1+i_o)} \prod_{t=1}^{n} (1+i_{t-1})} + \sum_{n=1}^{k} \frac{D_n \prod_{j=1}^{n} (1-DR_{a+j-1})}{\prod_{t=1}^{n} (1+i_t)} + \sum_{n=1}^{k} \frac{DR_{a+n-1}(\$1{,}000)}{\frac{\sqrt{1+i_n}}{(1+i_o)} \prod_{t=1}^{n} (1+i_{t-1})}$$

$$+ \sum_{n=1}^{k} \frac{DR_{a+n-1}(TD_n)}{\frac{\sqrt{1+i_n}}{(1+i_o)} \prod_{t=1}^{n} (1+i_t)} + \frac{TD_k \prod_{j=1}^{k} (1-DR_{a+j-1})}{\prod_{t=1}^{k} (1+i_t)} + \frac{CV_k \prod_{j=1}^{k} (1-DR_{a+j-1})}{\prod_{t=1}^{k} (1+i_t)}, \quad (2)$$

---

[13]The formula appears in Babbel (1978) and reflects corrections of some theoretical and technical errors encountered in the Kensicki (1974) version. The model assumes the policyowner will surrender the policy in a particular year, given survival up to that point.

While the model was designed for the valuation of a participating whole life insurance policy featuring a death benefit, a cash surrender value, and dividends, but not other options, the model may be extended to include options such as renewable and convertible clauses, policy loan values, reduced paid up insurance, and extended term insurance. For a presentation of a capital budgeting analysis of these options, see Longstreet and Power (1970). The model may also be reduced to fewer terms when it is used for the valuation of nonparticipating whole life and term insurance policies. For a nonparticipating whole life policy, the dividend and terminal dividend expressions are simply eliminated, and for a term policy, the dividend, terminal dividend, and cash value expressions are eliminated.

Later in this chapter, a probabilistic approach will be taken with respect to the uncertain timing of policy surrender. Aggregate lapse rates will be incorporated into the formula and k, the year of surrender, will no longer be viewed as the only year of policy surrender.

where

$E\left[NPV_k\right]$ is the Expected Net Present Value of the insurance policy, per thousand units of insurance in force, for the insured who plans to surrender the policy in year k;

P_n is the premium payable at the beginning of year n, per thousand units of insurance in force;

CV_k is the cash value at the end of year k, per thousand units of insurance in force;

k is the year of surrender;

DR_{a+n-1} is the conditional probability that an insured who survives to age a+n-1 will die before reaching age a+n where the insured's attained age as the policy goes into force is represented by the letter a;

i_t (or i_n) represents the opportunity cost for the time value of money in year t (or year n) and serves as a basis for determining the present value of any stream of future costs and benefits. In operational terms, it can be viewed as the after-tax interest rate selected by the individual representing his risk-free rate of return in year t (or n);

D_n is the dividend payable at the end of year n, per thousand units of insurance in force; and

TD_n is the terminal dividend at the end of year n, per thousand units of insurance in force.

Formula (2) shows the basic cash flows in a participating whole life policy: premiums, death benefit, cash surrender value, and dividends. The first expression represents the expected present value of the premiums payable. This outflow is weighted at each step to reflect the possibility that the premiums will not be paid due to death of

the insured.[14] P_n is discounted by $\frac{1}{(1+i_o)} \prod_{t=1}^{n}(1+i_{t-1})$ because premiums are payable at the beginning of the year.

The second expression represents the present value of the dividends that are expected to be received by the insured, weighted according to the probability that the insured will survive to receive them. This expression is discounted by $\prod_{t=1}^{n}(1+i_t)$ since dividends are receivable at the end of each policy year.

The third expression represents the death benefit, which is the amount of insurance in force (one thousand units) multiplied by the probability that it is received (i.e., the probability that the insured will die). The death benefit is discounted by $\frac{1+i_n}{1+i_o} \prod_{t=1}^{n} (1+i_{t-1})$ due to the availability of the death benefit uniformly throughout the year.

The fourth and fifth expressions represent the expected present value of the terminal dividend. The terminal dividend appears twice because this cash flow is payable to the insured when the policy matures by death or surrender. The discount factors in the expressions are based on the assumption that the policy will terminate due to death or surrender at the end of a policy year.[15]

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[14]Later in this chapter the possibility that the insured may not persist with the policy in a given year will be incorporated into the model. (See footnote 13.)

[15]Although the terminal dividend is not necessarily received at the end of the policy year in the event of death, the discount factor for

The final expression represents the expected present value of the cash surrender value for year k, the year of surrender. The probability that death will terminate the policy before the projected year of surrender is included in the numerator, and the denominator gives the discount factor associated with cash flows occuring in the year of surrender.

Since, for the purpose of this study, it is the life insurance policies of Brazil that are of concern, some of the expressions of equation (2) (along with their problems of estimation) can be eliminated. In Brazil, participating life insurance policies are not available; thus, the dividend and terminal dividend expressions may be removed. The currency unit for measuring insurance in force for a Brazilian policy may now be identified as the cruzeiro (Cr$). It will be convenient (but not necessary) to simplify equation (2) further by assuming that $i_t = i$ for all t. Then, by multiplying each term by a negative one, the equation for a level premium whole life policy reduces to:

$$E\left[NPC_k\right] = \sum_{n=1}^{k} \frac{P \dfrac{1}{(1-DR_{a-1})} \prod_{t=0}^{n-1} (1-DR_{a+t-1})}{(1+i)^{n-1}} - \sum_{n=1}^{k} \frac{DR_{a+n-1}(Cr\$1000)}{(1+i)^{n-\frac{1}{2}}}$$

$$- \frac{CV_k \prod_{t=1}^{k} (1-DR_{a+t-1})}{(1+i)^k}, \qquad (3)$$

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the "mortuary dividend" is calibrated as if the dividend is paid at the end of the policy year. The model assumes that if the mortuary dividend (whose value, unlike the death benefit, is not fixed in the contract) is paid out during a policy year, its nominal value will be reduced so that its present value will remain the same.

where $E\left[NPC_k\right]$ is the expected net present cost[16] of the insurance policy, per thousand cruzeiros of life insurance in force, for the insured who plans to surrender the policy in year k, and the rest of the variables are defined as before.

Inflation and the Cost of Life Insurance Policies

To be able to utilize equation (3) in examining the effect of anticipated inflation on the NPC of whole life insurance, the relationship between the nominal required rate of return, i, used by the individual in his discount factor, and the expected rate of inflation must be specified. Along the lines of Irving Fisher's (1930) monumental work on the inflation/interest rate issue, their relationship is specified as the following:

$$(1+i) = (1+r)(1+j), \tag{4}$$

where r is the real rate of return required on a riskless investment[17] and j is the annual rate of inflation expected to prevail during the period of concern to the insured. In the analysis that follows, it is assumed that the real required rate of return is expected to vary independently

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[16] As previously noted, anticipated inflation, as opposed to realized inflation, is the economically relevant factor in consumer valuation of life insurance. For an explanation, see Neumann (1967, 1969).

[17] Life insurance is considered to exhibit characteristics similar to a riskless investment when risk is defined in terms of the probability of payment default.

of the expected inflation rate.[18] Substituting equation (4) into (3) yields the following formulation:

$$E\left[NPC_k\right] = \sum_{n=1}^{k} \frac{P \dfrac{1}{(1-DR_{a-1})} \prod_{t=0}^{n-1} (1-DR_{a+j-1})}{(1+r)^{n-1}(1+j)^{n-1}} - \sum_{n=1}^{k} \frac{DR_{a+n-1}(\text{Cr\$}\,1000)}{(1+r)^{n-\frac{1}{2}}(1+j)^{n-\frac{1}{2}}}$$

$$- \frac{CV_k \prod_{t=1}^{k} (1-DR_{a+j-1})}{(1+r)^k(1+j)^k} . \tag{5}$$

---

[18] While this assumption is often employed in models of a world with perfect certainty, it has been the subject of considerable debate. The hypothesis that real discount rates are unrelated to the rate of inflation, of course, goes back to Fisher (1896, 1930). Mundell (1963) has argued that nominal interest-elastic demand or supply of money can lead to a reduction in the real rate of interest due to inflation expectations. However, Mussa (1975) and Enders (1976) separately have contended that a more appropriate macroeconomic model specification leads to conditions that do not necessarily result in a real interest rate decline with increased inflation expectations. Mundell's result or its inverse depends on whether cash balances and capital are complimentary or substitute assets.

The response of interest rates to inflation expectations has been examined by numerous economists. The research is almost entirely based on the response in one national economy, that of the United States. Hess and Bicksler (1975) have shown that the real rate of interest has not been stable, whereas Fama (1975, 1976) and Feldstein and Eckstein (1970) have found that the real rate of interest has been stable. The empirical justification for the use of this assumption in the Brazilian case is provided in Silveira (1973).

The propositions derived in this paper will assume an independent relationship, but will also hold true even if inflation expectations are positively or negatively related to the real rate of discount. The only condition that must be met is that if anticipated inflation rises, the nominal discount rate must also increase.

To determine the likely impact of a change in the expected rate of inflation on the cost of life insurance, the method of differential calculus may be employed such that

$$dE\left[NPC_k\right] = \frac{dE\left[NPC_k\right]}{dj}\,dj\,.$$

Returning to the level premium whole life policy (equation (5)) and differentiating[19]

$$dE\left[NPC_k\right] = P\,\frac{1}{(1-DR_{a-1})}\sum_{n=1}^{k}(1+r)^{1-n}(1-n)(1+j)^{-n}\prod_{t=0}^{n-1}(1-DR_{a+j-1})$$

$$-\ (Cr\$\,1000)\sum_{n=1}^{k}DR_{a+n-1}(1+r)^{\frac{1}{2}-n}(\tfrac{1}{2}-n)(1+j)^{-n-\frac{1}{2}}$$

$$-\ CV_k(1+r)^{-k}(-k)(1+j)^{-k-1}\prod_{t=1}^{k}(1-DR_{a+t-1})\quad dj. \qquad (6)$$

Here it is seen that for k = 1, 2, 3, . . . the first term will be a non-positive expression while the second and third terms (including their

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[19]In addition to assuming independence between the real required rate of return and the expected inflation rate, the model implicitly assumes that (1) mortality rates are independent of the inflation rate, and (2) policy terms are independent of the inflation rate. While the first assumption may be a close approximation to reality (except, perhaps, in the case of the fixed income recipient, whose anxiety level increases with inflation, thereby contributing to earlier death), the second assumption may not hold if markets are perfectly free to adjust policy terms in accordance with the rates of inflation. In Brazil, not only are the mortality assumptions permissible in actuarial calculations specified by the regulatory agencies, but the capitalization rate utilized

preceding signs) will be positive. The sum of these three expressions, whether positive or negative, specifies the direction of impact of a change in the expected rate of inflation on the expected net present cost of a given amount of life insurance coverage. If the sum is positive (negative), an increase in the expected rate of inflation will produce a rise (decline) in the expected net present cost of the policy. In Appendix A it is shown that for the insured who plans to surrender at the end of the first policy year, an increase in the expected rate of inflation will unambiguously lead to an increase in the expected net present cost of a term policy. It is also shown that if insurance is priced fairly (where present values of expected costs and benefits are equal), the same pattern will hold true for an insured who plans to surrender at the end of the second policy year. In general, however, determining the impact of expected inflation on the cost of life insurance will require more information with regard to the levels of costs in relation to benefits, the time horizon, and the mortality rates. When such information is provided, it was found for the Brazilian case that the expected net present cost of life insurance increases

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in the calculations is also regulated. This rate has changed only twice in the past 25 years remaining within a range of four to six percent, far below the rate that would be indicated by the endemic inflationary environment of Brazil.

with anticipated inflation for surrender in year one, but decreases with anticipated inflation for surrender thereafter.[20]

Cost Comparisons Over Time Under Differing Inflation Rate Assumptions

To illustrate the potential magnitude of the influence inflation may have on policy costs, two life insurance policies commonly available in Brazil were analyzed. The results of these analyses are presented in the tables that follow.

The insurance values of a term policy, calculated in accordance with the valuation equation (5) given earlier, are shown in Table 2.[21] A real rate of return required on the insurance investment equal to four percent ($r = .04$) was used in all calculations. Anticipated levels of inflation, j, selected for comparison include zero percent, five percent and thirty percent. Thus, recalling the Fisher condition

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[20]The author has extended the analysis to a leading participating whole life insurance policy offered in the United States, resulting in similar cost patterns.

[21]Because a term insurance policy produces no cash values, the final term of equation (5) may be ignored. In the equation, actual policy data obtained from one of the largest insurers in Brazil were used. The mortality assumptions employed in the estimations are given in Moura (1976). These are the most recently calculated mortality statistics available in Brazil. Unfortunately, the data reflect the death experience of group insurers only; using them here is at best an approximation of the mortality rates that apply to individual policies. Another assumption is that the representative individual possesses mortality probabilities similar to the population of individual life insurance policyholders as a whole.

$((1+i) = (1+r)(1+j))$, the discount factor $(1+i)$ becomes 1.04, 1.092 and 1.352, respectively.

In Table 2, column one indicates the policy year under consideration. Columns two, three and four give the expected present value of the future premiums due, calculated for the assumed anticipated inflation rates of zero, five and thirty percent, respectively.

Columns five, six and seven indicate the expected present value of the death benefit available during each policy year under each of the differing inflationary assumptions. Finally, in columns eight, nine and ten, the expected net present cost of the policy for each of the inflation rate assumptions is tabulated.

An examination of columns eight, nine and ten reveals that in year one, the expected net present cost of the term policy increases with increasing inflation. This is precisely in accordance with the *a priori* analysis.[22] In year two, however, this trend is reversed and the expected net present cost of the term policy decreases with increasing inflation. Upon extending the examination to the third, fourth and fifth years, it is found that at higher rates of inflation the expected net present cost of the term policy continues to be lower than that of the same policy without inflation.

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[22]See Appendix A.

TABLE 2

Five Year Term Insurance Policy: Expected Present Values*

Yr.	Present Value ∑Future Premiums			Present Value ∑Death Benefits			Expected Net Present Cost		
	j=.00	j=.05	j=.30	j=.00	j=.05	j=.30	j=.00	j=.05	j=.30
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	18.35	18.35	18.35	3.49	3.41	3.06	14.86	14.94	15.29
2	35.93	35.09	31.87	7.09	6.76	5.49	28.84	28.33	26.38
3	52.77	50.36	41.83	10.81	10.05	7.42	41.96	40.31	34.41
4	68.90	64.29	49.17	14.66	13.29	8.96	54.24	51.00	40.21
5	84.34	76.99	54.58	18.65	16.49	10.81	65.69	60.50	43.77

* Policy data for male, age 35; premium of Cr$ 18.35 per Cr$ 1000 insurance in force.

Next, the effects of differing anticipated rates of inflation upon the cost of an ordinary life policy over a twenty year period are viewed.[23] The valuation formula used is given by equation (5). In all of the comparisons, the required real rate of return, r, is constant throughout all time periods at four percent. Anticipated levels of inflation selected for comparison are the same as before.[24]

In Table 3, column one indicates the policy year under consideration. Columns two, three and four give the expected present values of the future premiums to be paid, as computed under the assumed rates of anticipated inflation of zero, five and thirty percent, respectively. Column

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[23] Policy data is from a policy marketed by a leading Brazilian company. Mortality assumptions are the same as before.

[24] While thirty percent may seem a high anticipated rate of inflation, this was the average realized rate of inflation in Brazil for the twenty year period from 1948 through 1967.

five lists the values of the death benefit in nominal terms for each of the twenty years under consideration. Columns six, seven and eight indicate the present values of these accumulated benefits under their respective inflation rate assumptions.

Column nine lists the cash values of the policy at the end of each year, guaranteed in the contract. The three columns that follow indicate the present values of these guaranteed cash values under each of the inflation rate assumptions, adjusted for mortality. Columns thirteen, fourteen and fifteen show the expected net present costs of the life insurance policy that relate to each of the inflation rate assumptions for the twenty possible years of policy surrender.

A comparison of the last three columns reveals that for year one, as with the term policy, the expected net present cost of the ordinary life policy increases with increasing inflationary expectations. In year two and thereafter, increasing inflationary expectations tend to decrease the net present cost of the policy.

These cost patterns may seem counter-intuitive at first blush, but can readily be explained through examining the interaction of two opposing forces deriving from the timing of cost and benefit flows, and the magnitudes of these flows. The timing of cost and benefit flows leads to an increase in the expected net present cost of an insurance policy, when inflation is introduced into the model. It will be noted that the benefit flows are always discounted at higher rates than the premium flow,

TABLE 3

Net Present Cost of a Whole Life Policy for Insured Male Age 35 with a Premium of Cr$30.51 per Cr$1000 Insurance in Force

Yr.	Present Value $\sum$ Future Premiums			Value Death Benefit	Present Value $\sum$ Death Benefits			Cash Sur. Value	Present Value Cash Surrender Value			Expected Net Present Cost		
	j=.00	j=.05	j=.30		j=.00	j=.05	j=.30		j=.00	j=.05	j=.30	j=.00	j=.05	j=.30
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
1	30.51	30.51	30.51	.11*	.11	.11	.09	-	-	-	-	30.40	30.40	30.42
2	59.74	58.35	53.00	3.82	3.71	3.46	2.52	-	-	-	-	56.03	54.89	50.48
3	87.74	83.75	69.57	4.10	7.43	6.75	4.45	8	7.03	6.07	3.20	73.28	70.93	61.92
4	114.55	106.91	81.77	4.41	11.27	9.99	5.98	18	15.15	12.46	5.30	88.13	84.46	70.49
5	140.22	128.03	90.76	4.76	15.26	13.19	7.21	31	24.96	19.55	6.72	100.00	95.29	76.83
6	164.89	147.28	97.38	5.13	19.39	16.35	8.19	45	34.65	25.86	7.18	110.85	105.07	82.01
7	188.39	164.81	102.25	5.55	23.69	19.48	8.97	58	42.72	30.35	6.80	121.98	114.98	86.48
8	210.86	180.78	105.83	6.01	28.17	22.59	9.60	72	50.68	34.30	6.21	132.01	123.89	90.02
9	232.33	195.31	108.46	6.51	32.83	25.67	10.10	87	58.50	37.71	5.51	141.00	131.93	92.85
10	252.84	208.53	110.39	7.07	37.70	28.73	10.50	102	65.48	40.19	4.75	149.66	139.61	95.14
11	272.43	220.55	111.81	7.68	42.79	31.78	10.82	118	72.28	42.26	4.04	157.36	146.51	96.95
12	291.12	231.48	112.85	8.36	48.12	34.82	11.08	134	78.26	43.58	3.36	164.74	153.08	98.41
13	308.94	241.40	113.61	9.10	53.69	37.85	11.29	150	83.47	44.26	2.75	171.78	159.29	99.57
14	325.92	250.40	114.17	9.92	59.53	40.87	11.46	167	88.47	44.68	2.25	177.92	164.85	100.46
15	342.08	258.56	114.58	10.82	65.66	43.89	11.60	184	92.71	44.59	1.81	183.71	170.08	101.17
16	357.45	265.95	114.88	11.81	72.09	46.91	11.71	202	96.71	44.31	1.45	188.65	174.73	101.72
17	372.06	272.64	115.10	12.90	78.84	49.93	11.80	220	99.97	43.62	1.16	193.25	179.09	102.14
18	385.92	278.69	115.26	14.20	85.99	52.97	11.87	239	102.95	42.77	.92	196.98	182.95	102.47
19	399.06	284.15	115.38	15.43	93.46	56.00	11.93	257	104.80	41.47	.71	200.80	186.68	102.74
20	411.50	289.07	115.47	16.88	101.32	59.03	11.98	277	106.78	40.25	.57	203.40	189.79	102.92

*If death occurs during the first year, only the premium is returned.

Calculations assume r=.04

since they occur later within each time period, producing a positive effect upon the expected net present cost of the policy.

The magnitudes of the cost and benefit flows exert a negative influence on the expected net present cost of an insurance policy when inflation is introduced into the model, if costs exceed benefits (the usual case). An example will serve to illustrate this point. If two unequal quantities are discounted equally in percentage terms, the larger of the two quantities will decline more in absolute terms. Thus, since costs are of larger magnitude than benefits, they will fall more rapidly in absolute terms with inflation, thereby decreasing the net present cost.

It appears in the case of the two Brazilian insurance contracts analyzed, the effect of inflation via the magnitudes of cost and benefit flows dominated the opposing effect of inflation deriving from the timing of the flows in all but year one.

In the foregoing methodology, uncertainty with respect to cash outflows and inflows was restricted to uncertainty about the time of death. (The nominal values of the premiums, death benefit, and cash values are fixed in the policy contract and thus are not subject to fluctuations.) The approach was designed to answer the question: "What is the expected net present cost of a life insurance policy that will be surrendered, given survival, at the end of year k?"

The model may be generalized by extending uncertainty not only to time of death, but also to time of policy surrender. The assumption

of the former model that the insured persists with the policy, given survival, up until year k, at which time he surrenders with probability of unity, is relaxed. Instead, policy surrender is viewed as possible in any year, with varying degrees of probability in each year.[25]

Rather than producing a matrix of net cost data according to year of surrender and expected rate of inflation, this probabilistic approach produces a vector of net cost data that vary only according to the expected rate of inflation. The probable duration of the contract is already taken into account in the calculations.

To illustrate this method, the ordinary life policy used in the previous example is recalled and the appropriate mortality and surrender rate assumptions are applied as follows:

(1) The premiums due outflow is weighted at each step by the probability that the premiums will be paid, that is, the joint probability that the insured survives and persists.

(2) The surrender cash values are weighted by the probability that they will be received, that is, the joint probability of surviving and persisting until the end of each year and surrendering at the end of the year.

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[25] The inclusion of persistence rates was suggested by Ferrari (1968) and Belth (1969). The persistence rates employed in the calculations that follow were based on the experience of one of the largest insurers in Brazil. According to the actuary of the company, the rates have not varied significantly over time, in spite of varying inflation rates.

(3) The expected death indemnification cash flow is equal to the face value of the policy multiplied by the probability of receiving it, which is given each year by the probability that the insured has survived and persisted up until that year and then dies during the year.

Having properly weighted the annual cost and benefit flows according to their probability of occurrence, their present values are calculated (for various rates of expected inflation) and summed over the possible life of the contract. The sum of expected benefits (in present cruzeiros) is subtracted from the sum of expected costs (in present cruzeiros) resulting in the expected net present cost (which by reversing the sign can be viewed as the expected net present value) of the policy.

In Figure 1 the expected present values of the cost and benefit flows per Cr$1000 insurance in force are shown, calculated for various inflation rate assumptions ranging from negative four percent to positive forty percent per year. The expected net present cost of the ordinary life policy over the same range of inflation rates is illustrated in Figure 2, which is a derivative of Figure 1. The curve in Figure 2 is derived by taking the difference between the top two curves in Figure 1. One notable characteristic of the particular ordinary life policy analyzed is that the expected net present cost reaches a maximum when stable prices are expected to prevail. As inflation is expected to rise, the expected net present cost of the life policy declines. This finding is

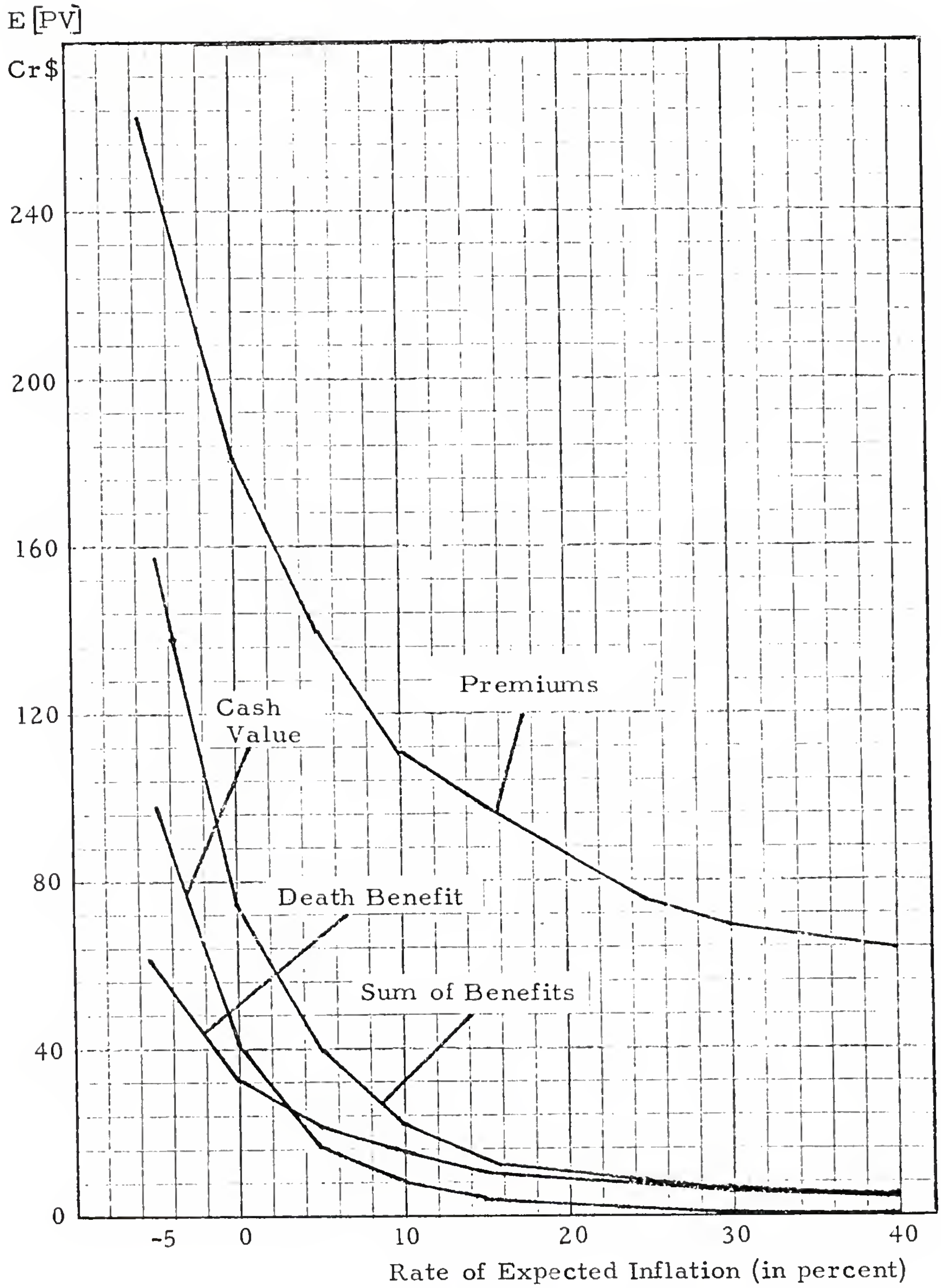

FIGURE 1: Expected Present Values of Life Insurance Cost and Benefit Flows under Differing Rates of Anticipated Inflation

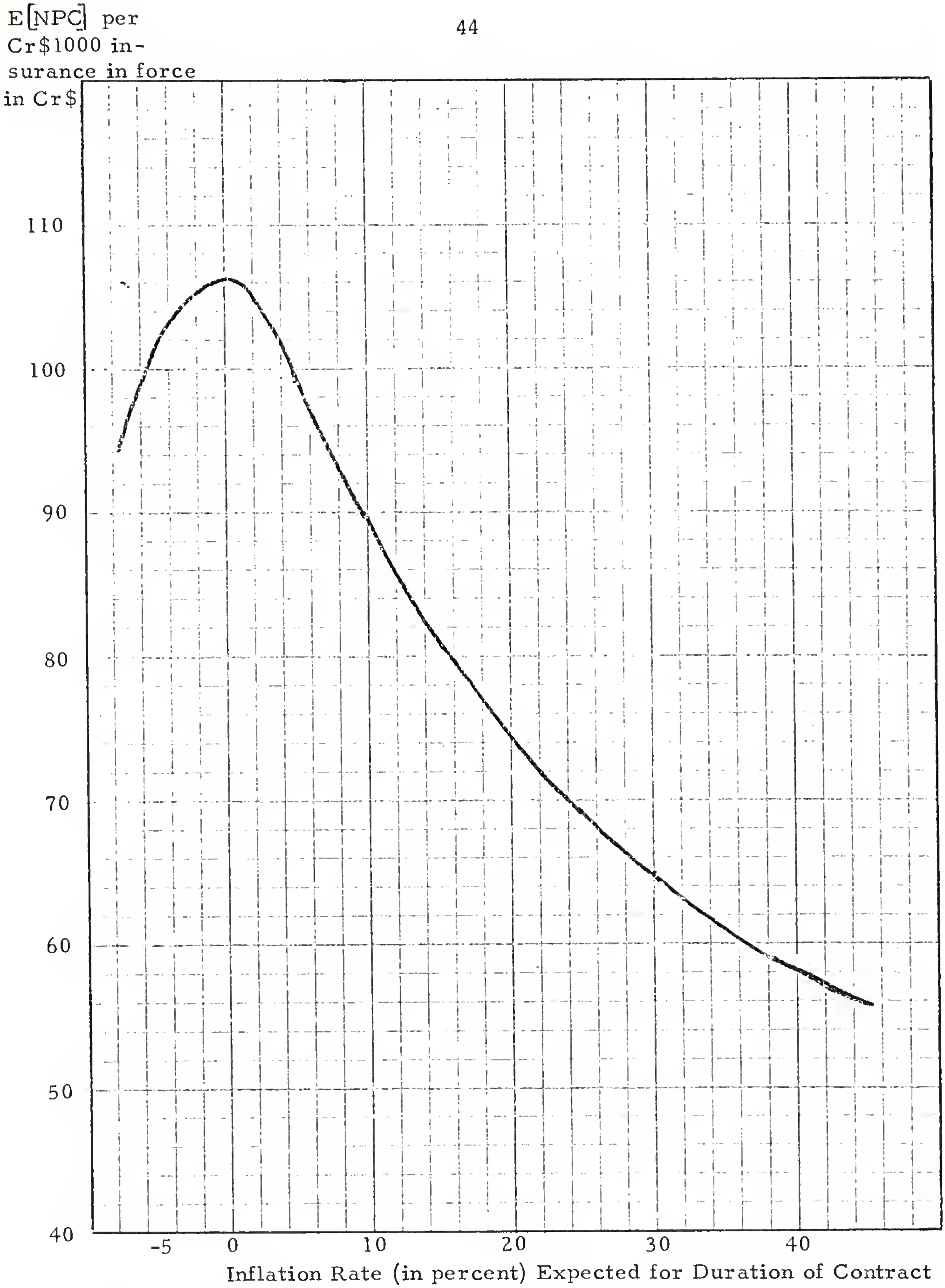

FIGURE 2: Expected Net Present Cost of a Whole Life Policy under Inflation

contingent, of course, upon the lapse rates used to weight the components in the streams of costs and benefits.[26]

The problem encountered in comparing expected (net) present costs of a life insurance policy under differing inflation rate assumptions, as performed in the analyses of this section, is that not only do the costs change, but the product also changes. Therefore, any comparison yields about as much useful information as comparing the cost of apples under one inflation rate assumption with the cost of peanuts under another inflation rate assumption, hoping thereby to infer the effect of inflation on the cost of apples. Under inflation, the real protection achieved through a policy purchase declines, while indemnification varies in real terms according to the date of death. Since the nominal terms of the policy remain the same, it is tempting to compare the effects of inflation on the cost of policies, rather than on life insurance protection. However, the policy is merely the vehicle through which the objective (i.e., protection) is sought. Thus, attention is more properly centered upon the effects of inflation on the cost of protection.

To demonstrate more concisely the problem associated with using the Expected Net Present Cost model, employed above, in

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[26]The lapse rates applicable depend upon the particular consumer involved in the purchase of the policy. In the calculations performed for the graph, aggregate lapse rates were used and it was assumed that a representative individual manifests probabilities of voluntary policy surrender similar to the group.

determining the effect of inflation on the cost of life insurance, the formula is given in simplified notation below:

$$E\left[NPC\right]/_{\substack{\text{per 1000 units}\\ \text{ins. in force}}} = E\left[PV(C)\right]/_{\substack{\text{per 1000 units}\\ \text{ins. in force}}} - E\left[PV(B)\right]/_{\substack{\text{per 1000 units}\\ \text{ins. in force}}} \quad (7)$$

In the above formula, C and B represent the cost and benefits associated with each one thousand currency units of insurance in force. The E and PV are expectations and present value operators, respectively. Unfortunately, the numeraire to which the benefits and costs are attached is a poor choice. The one thousand nominal units of insurance in force represent different levels of protection under differing inflation rate assumptions. Hence, such a model is inappropriate for use by a consumer free from policy illusion in determining the cost of life insurance.

Regrettably, many of the life insurance cost computation methods presently in use incorporate elements that tend to induce policy illusion on the part of the user. For example, the "Interest-Adjusted Method," which was developed to overcome defects in the "Traditional Method" and has been recommended by the Joint Special Committee on Life Insurance Costs (1970), and required by law in some states of the United States, makes explicit use of a nominally valued numeraire (one thousand units of insurance in force). Accordingly, when the discount factors are adjusted to reflect inflationary expectations, the change in the cost index that results will only indicate the

influence of inflation on the real cost of life insurance policies, and not on the real cost of protection available through the policies; hence the term "policy illusion."

No Illusion

It has been shown that if a consumer is befuddled with money illusion, partial money illusion, or policy illusion, he is likely to view the influence of inflation as producing unchanged, higher or lower costs, respectively, of life insurance.

To arrive at a costing methodology in which money illusion, in any of its forms, is absent, a good starting point is formula (7), copied below for convenience.

$$E[NPC]_{\text{/per 1000 units ins. in force}} = E[PV(C)]_{\text{/per 1000 units ins. in force}} - E[PV(B)]_{\text{/per 1000 units ins. in force}}$$

The nominal numeraire, which gives rise to policy illusion, can be eliminated in alternative ways. For the Expected Net Present Cost method, considered in detail earlier in the chapter, a simple and effective approach is to simply divide each term in the above formula by the last term that appears on the right hand side. The resulting equation is

$$\frac{E[NPC]_{\text{/per 1000 units ins. in force}}}{E[PV(B)]_{\text{/per 1000 units ins. in force}}} = \frac{E[PV(C)]_{\text{/per 1000 units ins. in force}}}{E[PV(B)]_{\text{/per 1000 units ins. in force}}} - \frac{E[PV(B)]_{\text{/per 1000 units ins. in force}}}{E[PV(B)]_{\text{/per 1000 units ins. in force}}} \quad (8)$$

which simplifies to

$$\frac{E\left[NPC\right]}{E\left[PV(B)\right]} = \frac{E\left[PV(C)\right]}{E\left[PV(B)\right]} - 1 \, . \tag{9}$$

Note that in the above expression, the numeraire of one thousand nominal units of insurance in force has been cancelled out of all the terms. What remains is the expected net cost (in present value terms) per unit of expected benefits (also in present value terms). It is noted that the above expression differs from the expected cost-benefit ratio by an amount equal to unity. In economic terms what has been done is that the price of insurance has been deflated by the actuarially fair price of insurance. This is appropriate because a fair price is "costless" to the individual in the sense that his expected expenditure is equal to his expected benefits (Ehrlich and Becker, 1972).

Three problems immediately arise in operationalizing the general formulation given by (9). First, the matter of what should be considered a cost and what should be considered a benefit of life insurance. This problem was not encountered in the horizontal outlay of the Expected Net Present Cost method, but the vertical nature of the cost-benefit ratio raises questions of what properly constitute the components of the ratio. Subtracting a particular item from the cost component rather than adding the item to the benefit component will affect the ratio. Although insurance premiums and death benefits are unquestionably costs and benefits, respectively, of a life insurance policy, other items such as cash surrender values, and dividends (where available) can

either be viewed as reductions in the cost or as additional benefits (Belth, 1969). Still another approach is to remove such elements altogether from insurance cost calculations and to determine separately the costs of these benefits.[27]

A second problem arises if uncertainty is extended to include the inflation rate, as well as the intraperiod moment of death. Up to this point it has been assumed that if death occurs in a given period, it occurs, on the average, about midway through the period. Another implicit assumption was that there is no uncertainty regarding rates of inflation that will prevail in the future. Relaxing either of these assumptions will result in an additional source of randomness in the values of cost and benefit flows. Relaxing both assumptions will serve to reinforce the possible deviations in realized flows from expected flows.

For any insured who is concerned with more than just the first moments of the cost and benefit distributions, the existence of uncertainty will affect the consumer's perception of the cost of insurance protection. For example, if the insured is risk averse with respect to the value of his bequest in the event of death, the fact that the expected present value of his bequest is adequate may not quell his concern. For such an insured, the unit of account — expected present value units of

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[27]Examples of this approach can be found in Levy and Kahane (1970), Ferrari (1968), Schwarzschild (1967, 1968), Linton (1964), and Belth (1961, 1966, 1968).

benefits — will no longer serve as an adequate numeraire. Expecting to leave "on the average" a given real-valued bequest in the event of death is not the same as leaving with certainty the same real valued bequest in the event of death. Since death is a once in a lifetime occurrence to the (typical) insured, the law of large numbers is of little consolation to the risk averse individual or to the heirs for whom he is seeking to ensure financial security.

There is a third problem in using the model. Different expected cash flows may elicit differing discount rates, even in a model such as the one presented here where uncertainty is limited only to survival (Hirshleifer, 1970). If the model is used in isolation as a decision criterion — to buy or not to buy insurance — it should be recognized that, even though the expected cash flows associated with insurance may not exhibit covariance with other financial assets in the insured's investment portfolio, the death benefit has negative covariance with the return on human capital portion of the insured's portfolio and thus may elicit a lower or even negative discount rate.[28]

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[28]Whether or not differing discount rates may properly be applied to separate cash flow streams that are part of a "package" is a debatable issue. Arditti (1974) has demonstrated that the procedure of using a discount factor for cash flows exhibiting complete certainty different from that applied to uncertain flows is appropriate. Other cash flows, he contends, should be discounted by a similar risk-adjusted rate, because they are not separable. Whether these conclusions are appliable to the life insurance product is debatable. The policy can be arranged to include any desirable cash flow provisions, and is cancellable at any moment.

Having identified three potential problems in using formula (9) for the valuation of life insurance, we proceed to deal with each. In the remainder of this chapter, the first problem will be resolved by treating the surrender cash value as a benefit, rather than a reduction in cost; hence, it is included in the denominators of each term in formula (9). The second and third problems both evoke individual utility considerations in their resolution; these problems are deferred until the next chapter, where they are handled in the more powerful analytical framework of Time-State Preference. In defense of the methodology employed in this chapter, there is merit in devising a cost index that is independent of specific utility functions. The index is then universal in application, and provides an unbiased measure of insurance costs in expected monetary (present) values. This information can then be evaluated in conjunction with other factors (e.g. other moments of the cost and benefit distributions) which may receive varying levels of importance, in accordance with the individual who is appraising the insurance product.[29]

To determine the impact of anticipated inflation upon the cost of life insurance (as measured by the expected net present cost per unit of expected benefit in present value) the differential of equation (9) may be

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[29]The Expected Monetary Value method is discussed at length in Mao (1969), and is evaluated in conjunction with the variance associated with the resulting value.

taken as follows:

$$d \frac{E\left[NPC\right]}{E\left[PV(B)\right]} = \frac{d \dfrac{E\left[PV(C)\right]}{E\left[PV(B)\right]} dj}{dj}$$

If the derivative of the cost-benefit ratio is positive, a rise in anticipated inflation will cause an increase in the expected net present cost, per (present-valued) unit of expected benefit. In Appendix B it is demonstrated mathematically that as long as an increase in anticipated inflation is associated with an increase in the discount rate, the sign of the derivative of the cost-benefit ratio is positive; hence, an increase in expected inflation will be associated with an increase in the cost of life insurance.

To illustrate the potential magnitude of the effect of inflation on life insurance cost, actual policy data may be substituted into formula (9). For expository purposes, the same ordinary life policy analyzed in the previous section will be subjected to the analysis procedure specified by formula (9). Returning to Figure 1, the procedure simply involves selecting an inflation rate and dividing the expected cost by the expected benefits (top two curves) at that point, and subtracting one from the resulting quotient. Below are presented the expected net present costs, per unit of benefit, at various levels of expected inflation.

Rate of Inflation Anticipated (r=4%)	PV Expected Cost / PV Expected Benefits	-1	Expected Net Present Cost per Unit of Benefit
- 3.85%	237.83/136.76	-1	0.74
0.0 %	181.53/75.42	-1	1.41
+ 5.0 %	138.24/38.97	-1	2.55
+ 10.0 %	112.64/22.84	-1	3.93
+ 15.0 %	96.35/14.92	-1	5.46
+ 25.0 %	77.46/8.19	-1	8.46
+ 30.0 %	71.56/6.55	-1	9.93
+ 40.0 %	63.34/4.68	-1	12.53

An unambiguous upward trend in expected cost is shown in the above figures for life insurance protection as anticipated inflation rises. In particular, a rise in anticipated inflation from zero to forty percent is shown to be associated with a rise in expected net present cost per unit of benefit of 791 percent. Although the precise magnitudes of cost increases shown are obviously dependent upon the conditions of the particular policy analyzed, including the surrender and mortality rate assumptions utilized, the upward trend is not subject to these qualifications. Increasing rates of inflation always lead to higher costs of life insurance protection available through conventional contracts. The only exception to this rule is when the nominal terms of the insurance contract (specified premiums and benefits) adjust to fully compensate the consumer for changes in inflation rates. Unfortunately, this is

seldom the case. Life insurance industries are often constrained by governmental regulators, and are generally very slow to adjust to inflation.[30]

Maintaining Real Values

By reducing the real value of protection provided by life insurance, inflation thwarts the primary design of the insurance contract, i.e., to provide protection against the perils of premature death and of outliving one's earning capacity. Thus, it may be argued that a more meaningful measure of inflation's impact upon the valuation of life insurance will reflect the cost of achieving a desired pattern of real protection over time (as opposed to simply providing for a desired expected real value of protection). Whether the insured desires increasing, decreasing, constant, or some other pattern of real protection over time is a behavioral question and beyond the scope of this study.

For expository purposes, it will be assumed in the following two subsections of this paper that the insured wishes to maintain a constant real level of protection over time against the peril of premature

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[30] See explanation in footnote 19. The blame does not necessarily lie with the regulators. Uncertainty regarding the future interest rates makes it irresponsible for a regulator to permit the full expected rate of interest (and inflation) to be incorporated in the actuarial calculations, because a negative deviation between the expected and realized rates could create problems of insolvency. Furthermore, the insurance industry may deem it more profitable not to adjust their rates to incorporate interest and inflation rate expectations, especially if insurance demand is price inelastic.

death.[31] Protection from outliving earning capacity will be adjusted in nominal terms according to steps taken by the insured aimed at maintaining constant a real level of protection against the peril of premature death; this adjustment, however, will not necessarily result in maintaining constant a real level of protection against the peril of outliving earning capacity.

Additional Insurance Purchases

The insured may seek to maintain a real level of protection in several ways. Insurance policies are often equipped with provisions for increasing coverage periodically through purchases of paid-up additions, term riders, etc. Another option available to the insured is to periodically purchase additional policies to compensate for the value of the protection eroded in his existing policy(ies). If index-linked policies are available, still another option is possible for the insured.

It is assumed in the following analysis that the insured seeks to maintain a real level of protection through annual purchases of additional life insurance policies (hereafter abbreviated AAP). Whether these purchases are made to cover real protection reduced by realized

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[31]The analysis is only slightly more difficult for alternative paterns of desired protection. It should be noticed, moreover, that in order to achieve the goal of maintaining constant a desired real level of protection, policy values would need to adjust constantly to compensate for changes in the purchasing power of money. Thus, constant real levels of protection will not actually be maintained, but will only be approximated by strategies listed in this section.

or anticipated inflation is unimportant for the present analysis as both yield similar results. In this section it will be assumed that adjustments are made each year to recapture protection value lost due to realized inflation.[32] As with the previous analysis, policy fees incurred will be ignored.[33] However, later in this chapter the direction of their effect under inflation, when included in the analysis, will be shown.

Examined first is the effect of inflation upon an insured who seeks a constant real level of protection through term insurance. Term insurance offers protection against the peril of premature death, but it offers no cash values which could be used to offset expenses incurred after earning capacity is outlived. Hence, it may be inferred that the insured seeks only to protect himself from the first peril through the medium of life insurance if he patronizes exclusively term insurance products. At the end of each year, the insured increases the coverage of his death benefit by the amount of inflation realized during the

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[32]In a world of perfect certainty, anticipated rates of inflation will be realized. Alternatively, the insured may opt to adjust his insurance coverage so as to obtain his desired real level of protection at the end of each period. This strategy entails carrying (and paying for) more real protection than is desired during the period. The results of either strategy yield similar effects upon the net present costs.

[33]In addition to ignoring policy fees, it is further assumed that life insurance coverage can be purchased in any amount at a fixed percentage cost per Cr$ 1000 coverage. In reality, there are restrictions on the minimum amount purchasable per policy (and the amounts insurable are usually multiples of Cr$ 1000). Furthermore, a policyholder's health may render him ineligible to continually purchase new life insurance policies.

period, which shall be designated j. This adjustment will incur premium costs to the insured that will rise by the same percentage that coverage rises, namely j.

The effect of these adjustments upon the expected net present cost of the insurance can be determined by returning to equation (9) (omitting the cash surrender value expression, which is not relevant in term insurance policies) and making the indicated revisions. To facilitate the presentation notationally, the following abbreviated notation will be employed:

$s_n = \prod_{t=1}^{n} (1-DR_{a+t-1})$ = the probability of survival through the end of year n;

$s_{n-1} = \frac{1}{(1-DR_{a-1})} \prod_{t=0}^{n-1} (1-DR_{a+t-1})$ = the probability of survival through the end of year n-1;

$d_n = DR_{a+n-1}$ = the probability of death in year n, given survival up through the beginning of the year;

$R_t = (1+r_t)$ = the real discount factor; and

$J_t = (1+j_t)$ = the inflationary adjustment to the discount factor.

Substituting these expressions into equation (9) yields:[34]

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[34]Unlike the analysis of insurance encountered earlier in this paper, the analyses of this section do not assume level premiums; on the contrary, P_n reflects the premium required to insure during each year n, with no surpluses in earlier years carried forward to cover increasing mortality rate expenditures in later years. If the insured may only purchase level premium term policies of (say) k years, the strategy of additional policy purchases would serve to increase the net present costs even further by increasing the real value of premiums charged.

$$\frac{E[NPC]}{E[PV(B)]} = \frac{\displaystyle\sum_{n=1}^{k} \frac{P_n s_{n-1}}{\frac{1}{R_o J_o} \prod_{t=0}^{n-1} (R_t J_t)}}{\displaystyle\sum_{n=1}^{k} \frac{d_n(Cr\$\,1000)}{\frac{R_n^{\frac{1}{2}} J_n^{\frac{1}{2}}}{R_o J_o} \prod_{t=0}^{n-1} (R_t J_t)}} - 1, \tag{10}$$

which, when revised to include annual additional policies purchased, becomes

$$\frac{E[NPC]}{E[PV(B)]} = \frac{\displaystyle\sum_{n=1}^{k} \frac{P_n s_{n-1} \frac{1}{J_o} \prod_{t=0}^{n-1} J_t}{\frac{1}{R_o J_o} \prod_{t=0}^{n-1} (R_t J_t)}}{\displaystyle\sum_{n=1}^{k} \frac{d_n(Cr\$\,1000) \frac{1}{J_o} \prod_{t=0}^{n-1} J_t}{\frac{R_n^{\frac{1}{2}} J_n^{\frac{1}{2}}}{R_o J_o} \prod_{t=0}^{n-1} (R_t J_t)}} - 1. \tag{11}$$

Equation (11) simplifies further by noting that most of the terms denoting inflation adjustments cancel out of the ratio, leaving

$$\frac{E[NPC]}{E[PV(B)]} = \frac{\displaystyle\sum_{n=1}^{k} \frac{P_n s_{n-1}}{R_o \prod_{t=0}^{n-1} R_t}}{\displaystyle\sum_{n=1}^{k} \frac{d_n(Cr\$\,1000)}{\frac{R_n^{\frac{1}{2}} J_n^{\frac{1}{2}}}{R_o} \prod_{t=0}^{n-1} R_t}} - 1. \tag{12}$$

If the AAP strategy were able to completely overcome the value erosion produced by inflation, the valuation equation would not retain any adjustments for inflation, since adjustments for payments and

benefits would exactly be canceled by adjustments in the discount rates. However, as shown in formula (12) above, the AAP strategy does result in the retention of one of the inflationary adjustment terms: $J_n^{\frac{1}{2}}$ remains in the denominator of the (lower) death benefit expression. Therefore, for any positive J_n, the expected cost of the strategy will be increased. Hence it may be concluded that the expected net present cost, per unit of benefit, of a term insurance policy under stable price level conditions will always be less than the expected net present cost of maintaining a real level of protection through additional policy purchases under inflationary conditions.

Next, the same strategy (AAP) is examined, but with regard to investment life insurance (i.e., policies exhibiting cash values). This will require the inclusion of the cash surrender benefit added to the denominator of equation (12). Since the effect of inflation on the premium and death benefit cash flows has already been examined, only the surrender cash value expression remains to be analyzed. It is noted *a priori* that if inflation has either a neutral or negative effect upon the value of this component, the overall effect of inflation on an investment life insurance policy will be one of increasing its expected net present cost. It is helpful to observe at the outset that the cash-value component will remain neutral to inflation in real terms only if the AAP strategy achieves adjustments in the nominal surrender value that exactly

compensate for inflation. More formally, CV_k must rise by $\prod_{t=1}^{k} J_t$ so that:

$$\frac{CV_k s_k \prod_{t=1}^{k} J_t}{\prod_{t=1}^{k} R_t J_t} = \frac{CV_k s_k}{\prod_{t=1}^{k} R_t}. \tag{13}$$

Unfortunately, the AAP strategy does not achieve this result. The nominal cash value expected to be received upon surrender in year k is given by

$$s_k\left[CV_k + CV_{k-1}j_1 + CV_{k-2}j_2j_1 + \ldots + CV_1 j_{k-1}J_1 \ldots J_{k-2}\right], \tag{14}$$

which reduces to

$$s_k\left[CV_k + \sum_{n=1}^{k-1} CV_{k-n} j_n \prod_{t=1}^{n-1} J_t\right]. \tag{15}$$

Whether (15) is greater than, equal to, or less than the numerator of (13) determines if inflation will subtract from, render unchanged, or add to the expected net present cost per (present-valued) unit of benefit of an investment life insurance policy. The l.h.s. numerator of (13) can be expanded and rewritten as

$$s_k\left[CV_k + CV_k j_1 + CV_k j_2 J_1 + \ldots + CV_k j_{k-1} J_1 \ldots J_{k-2} + CV_k j_k J_1 \ldots J_{k-1}\right]. \tag{16}$$

Clearly $CV_k > CV_{k-1} > CV_{k-2} \cdots$ so that for positive j's (16) will always

exceed (14).[35] The conclusion follows that the effect of inflation on the cash-value component contributes to an increase in the cost of a life insurance policy with a savings component (as measured by the expected net present cost per unit of benefits). Once again, stable price levels result in a higher valuation to life insurance than rising price levels.

Index-Linked Policies

Attention now turns to index-linked life insurance policies, as a strategy to mitigate the adverse effects of inflation upon the value of life insurance. In particular, the analysis focuses upon the Brazilian system of indexing which was authorized for insurance contracts beginning November 21, 1966.

Brazilian life insurance contracts are offered in a variety of "index-linked" packages. A contract may be opted for in prefixed indexes, or may be periodically adjusted for realized inflation rates. Policies adjusted for realized inflation rates may feature partial or full adjustments of premiums, death benefits and cash values.[36] Of special interest are the policies fully adjusted for realized inflation, since these will best help the insured maintain a real level of protection.

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[35] Cash values increase in nominal terms over the life of the contract.

[36] For instance, a policyowner may elect to have his policy premiums and benefits adjusted for 50, 75 or 100 percent of the index utilized in "correcting" the values of life insurance contracts.

In Brazil a fully indexed[37] life insurance policy has the following characteristics:

(1) Annual premiums are adjusted at the beginning of each year in accordance with the rate of inflation realized during the course of the preceding year.[38]

(2) Death benefits are adjusted at the beginning of each year to recover the protection lost due to the erosion of inflation from the previous year. The effect of adjusting the death benefit on an annual basis, (where the death benefit is available uniformly throughout the year), differs from the effect of adjusting premiums due on an annual basis, since premiums are an annual cash flow. In an inflationary environment, the latter maintains the real value of the cash flows, whereas with the former the real protection is being constantly eroded. In other words, the costs are maintained constant in real terms but the death benefits are losing value throughout the year. For instance, if an insured dies ten months into the policy year, his beneficiaries receive the same amount in nominal terms as if he had died at the beginning of the year. Where inflation rates are high, there can be a substantial difference between the real value of the payments.

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[37]Although a policy may be 100 percent linked to the index, the index may not necessarily reflect 100 percent of the realized inflation. For an explanation of the formulation and modification of the indices used for life insurance contracts, see Appendix C.

[38]Because of difficulties of a technical nature in preparing the indices, the value actually used may reflect another period (see Appendix C).

(3) Surrender cash values, for policies that feature them, are tied in fixed nominal terms to the value of insurance in force. Because the nominal value of insurance in force is only readjusted at the beginning of each year, and thus, the cash value received at the end of the year will not reflect any adjustment for inflation occurring during the year.

The effects of inflation are easily seen when the above characteristics are reduced to mathematical form. The formula for the expected net present cost per (present-valued) unit of benefit of an index-linked term policy is precisely the same as formula (12), where the insured purchases new policies on an annual basis to compensate for the effects of inflation realized during each year. Note that both strategies result in higher insurance costs than those obtained where price levels are stable. (If prices were stable, the appropriate valuation formula would be identical to formula (12) after the $J_n^{\frac{1}{2}}$ term is removed.)

The equivalent formulas for determining expected net present cost ratios may lead one to conclude that an indexed term policy has equal merits to a simple annual routine of additional policy purchases. By relaxing some of the assumptions, however, it becomes readily apparent that there are significant differences:

(1) Earlier it was assumed that the insured could purchase additional term insurance policies at will. In practice, the future state of the insured's health is uncertain, and the insured may not be able to always qualify for new insurance policies.

(2) Upon relaxing the assumption that the insured could purchase additional term insurance in any quantities, the insured may be forced to purchase either more or less coverage than he desires. If the minimum amount of insurance purchasable is high (say Cr$ 250,000) this may lead to extended periods where the insured is under-protected.
(3) Medical examinations are often required for new policy purchases, and this recurring cost can amount to a substantial sum over time.
(4) Policy fees and other expenses are often fixed costs, independent of the level of coverage desired. Buying many small policies periodically will incur these costs more frequently, if they are added directly to the premium charged. (On the other hand, these costs added to the premium would automatically increase with index-linked contracts in accordance with the rate of inflation. It can be presumed, however, that costs incurred by taking the first approach would be greater unless the inflation rate is in excess of 100 percent.)
(5) Making annual new insurance policy purchases involves a commitment of time and energy above that involved in purchasing an index-linked policy.

When the above factors are taken into consideration, it is concluded that in an inflationary environment, an index-linked term insurance policy will result in lower expected net present costs, per unit of benefit, than will a series of term policy purchases aimed at maintaining a real level of protection.

Will index linking an investment policy produce similar benefits? The analysis for the premium and death benefit cash flows is the same as that given for a term policy. That leaves only the surrender cash value to investigate.

Previously it was noted that to remain neutral to the effects of inflation, the nominal value of the surrender cash values must increase at the same rate as inflation (as per equation 13). Because cash values in Brazilian index-linked policies are only adjusted to recapture the real value lost to realized inflation at the beginning of the next year, their expected present values are given by

$$\frac{CV_k s_k \frac{1}{J_o} \prod_{t=1}^{k} J_{t-1}}{\prod_{t=1}^{k} (R_t J_t)} = \frac{CV_k s_k}{J_k \prod_{t=1}^{k} R_t} \,. \tag{17}$$

This formulation clearly shows that under positive rates of inflation ($J_k > 0$), the expected present value of the cash value will be less than if there were no inflation. Expanding the numerator of the left side enables a more precise comparison of the cash values under the various alternatives. Formula (18) shows the nominal cash value expected for an indexed policy. Formulas (14) and (16), representing the expected cash values obtained through annual additional policy purchases and through a hypothetical policy which maintains inflation neutrality, respectively, are reproduced here to allow for convenient comparison.

$$s_k \left[CV_k + CV_k j_1 \quad + CV_k j_2 J_1 \quad + \cdots + CV_k j_{k-1} J_1 \cdots J_{k-2}\right] \tag{18}$$

$$s_k \left[CV_k + CV_{k-1} j_1 + CV_{k-2} j_2 J_1 + \cdots + CV_1 j_{k-1} J_1 \cdots J_{k-2}\right] \tag{14}$$

$$s_k \left[CV_k + CV_k j_1 \quad + CV_k j_2 J_1 \quad + \cdots + CV_k j_{k-1} J_1 \cdots J_{k-2} + CV_k j_k J_1 \cdots J_{k-1}\right] \tag{16}$$

Because these nominal cash values are all discounted by the same factor, namely $\prod R_t J_t$, a direct comparison of the above values is admissible.

A comparison of the above terms reveals that an inflation neutral policy will return ((16)-(18)) or $s_k CV_k j_k J_1 \cdots J_{k-1}$ in nominal expected cash value more than a Brazilian "fully indexed" policy will return. However, a Brazilian "fully indexed" policy will return ((18)-(14)) or

$$s_k \sum_{n=1}^{k-1} (CV_k - CV_{k-n}) j_n \prod_{t=1}^{k-2} J_t$$

more than would be achieved through purchases of additional policies. (It is interesting to note here that removing the CV_k term at the beginning of each of the above three expressions gives in nominal terms the additional cash value that each alternative is expected to return over the purchase of a single, non-adjusting policy.) It is concluded that an index-linked investment life insurance policy will yield superior returns (or result in lower expected net present costs) than those obtained through additional policy purchases, not only for the five

reasons stated earlier in the term policy section, but also because an index-linked policy will result in higher cash values.

In Table 4 the mathematical determination of the present values of the insurance cost and benefit flows is summarized, according to the mode of policies purchased.[39] Note that while in the first three rows the present values of the costs are maintained in real terms, the present values of the benefits are not, except when there is no inflation.

A graphical representation (Figure 3) follows the summary table, in which the present values of actual policy data (as determined by the expressions in Table 4) are combined with mortality and persistence data characteristic of the insurance industry. The resulting net present cost ratios for a typical nonindexed whole life policy and an index-linked whole life policy are represented by the upper and lower curves of the figures, respectively. It is worthwhile to note the dimensions of the cost changes as inflation increases. Neither the indexed nor the nonindexed policies are insulated fully from the effects of inflation, as both curves slope upward to the right. However, the index-linked policy exhibits a much more gradual slope upward, indicating less cost sensitivity to inflation. The data from which the curves were plotted is provided in Tables 5 and 6.

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[39]The formula components shown in Table 4 do not include the s_k and s_{n-1} multipliers in the premiums and cash value columns, because each of the insurance modes uses the same survival rates; however, this omission was only for the purpose of convenient comparisons, and the factors are included in calculations used in constructing the figures.

TABLE 4

Summary Table of Present Values of Life Insurance Cash Flows

INSURANCE MODE	PREMIUMS (Present Value)	DEATH BENEFIT (Present Value)	CASH VALUE (Present Value)
Inflation-Proof Policy (equivalent to purchase of a single non-adjusting policy without inflation)	$\boxed{2}$ $-\sum_{n=1}^{k} \frac{P_n}{\pi(1+r_{t-1})}$	$\boxed{1}$ $+\sum_{n=1}^{k} \frac{DR_{a+n-1}(\text{Cr\$ } 1.000)}{\sqrt{1+r_n}\,\pi(1+r_{t-1})}$	$\boxed{1}$ $+\frac{CV_k}{\pi(1+r_t)}$
Brazilian Fully Indexed Policy	$\boxed{2}$ $-\sum_{n=1}^{k} \frac{P_n}{\pi(1+r_{t-1})}$	$\boxed{2}$ $+\sum_{n=1}^{k} \frac{DR_{a+n-1}(\text{Cr\$ } 1.000)}{\sqrt{1+r_n}\sqrt{1+j_n}\,\pi(1+r_{t-1})}$	$\boxed{2}$ $+\frac{CV_k}{(1+j_k)\,\pi(1+r_t)}$
Additional Annually Purchased Policies	$\boxed{2}$ $-\sum_{n=1}^{k} \frac{P_n}{\pi(1+r_{t-1})}$	$\boxed{2}$ $+\sum_{n=1}^{k} \frac{DR_{a+n-1}(\text{Cr\$ } 1.000)}{\sqrt{1+r_n}\sqrt{1+j_n}\,\pi(1+r_{t-1})}$	$\boxed{3}$ $+\frac{CV_k + \sum_{n=1}^{k-1} CV_{k-n} j_n \prod_{t=1}^{n-1}(1+j_t)}{\pi(1+r_t)(1+j_t)}$
Single Non-Adjusting Policy With Inflation	$\boxed{1}$ $-\sum_{n=1}^{k} \frac{P_n}{\pi(1+r_{t-1})(1+j_{t-1})}$	$\boxed{3}$ $+\sum_{n=1}^{k} \frac{DR_{a+n-1}(\text{Cr\$ } 1.000)}{\sqrt{1+r_n}\sqrt{1+j_n}\,\pi(1+r_{t-1})(1+j_{t-1})}$	$\boxed{4}$ $+\frac{CV_k}{\pi(1+r_t)(1+j_t)}$

Ranking according to present values: 1, 2, 3, 4, where 1 indicates the highest present value (or least negative), higher numbers indicate lower present value. The ranking number is located in upper right-hand corner of each box.

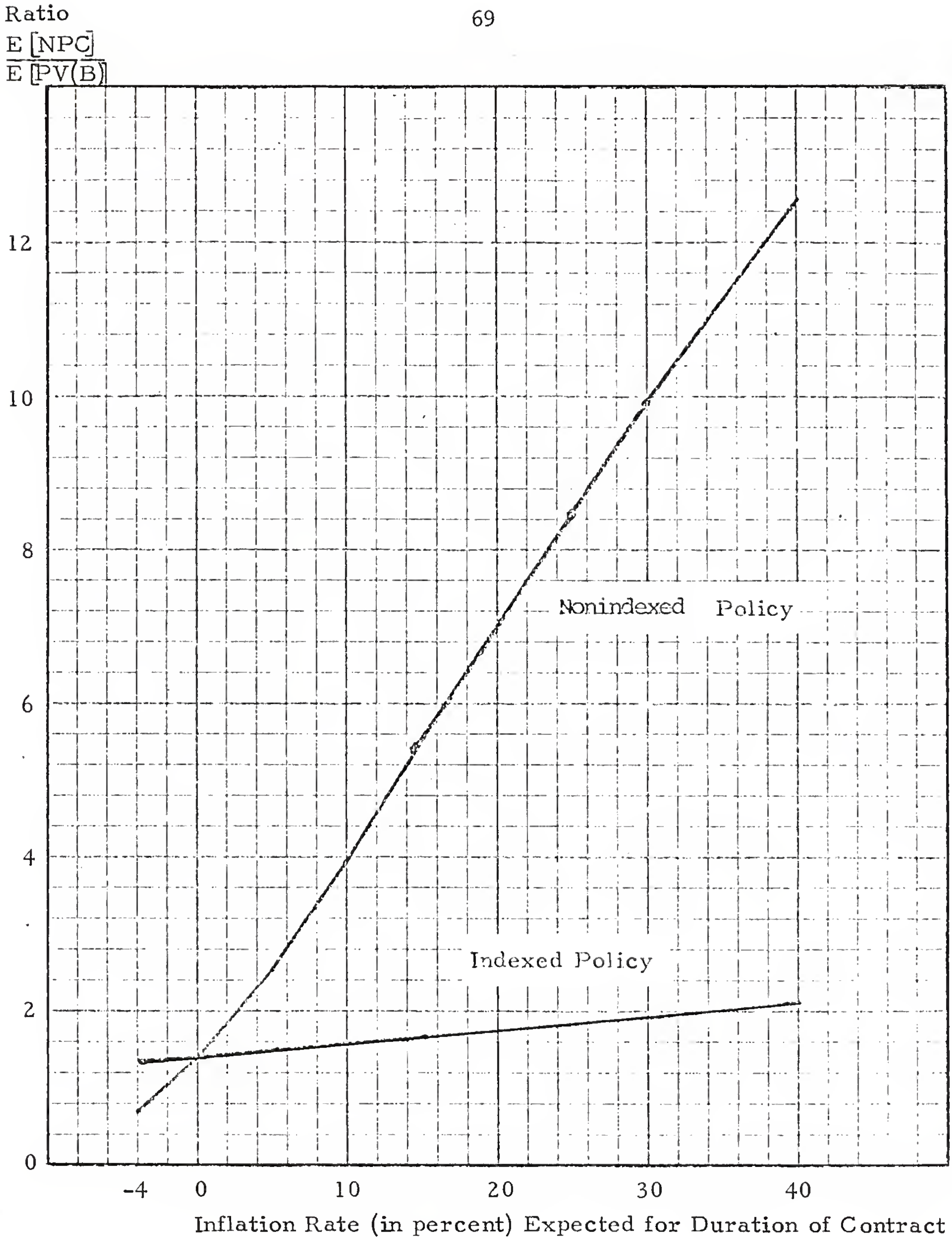

FIGURE 3: Expected Net Present Cost per (PV) Unit of Expected Benefits

TABLE 5

Nonindexed and Indexed Policy Values

Expected Rate of Inflation*	E [PV] Premiums	E [PV] Indemnification	E [PV] Cash Surrender Value
Nonindexed Policy			
-3.9%	237.83	52.86	83.90
0.0%	181.53	34.26	41.16
5.0%	138.24	21.34	17.63
10.0%	112.64	14.52	8.32
15.0%	96.35	10.60	4.32
25.0%	77.46	6.61	1.58
30.0%	71.56	5.48	1.07
40.0%	63.34	4.07	.61
Indexed Policy			
-3.9%	181.53	34.94	42.81
0.0%	181.53	34.26	41.16
5.0%	181.53	33.43	39.20
10.0%	181.53	32.67	37.42
15.0%	181.53	31.95	35.79
25.0%	181.53	30.64	32.93
30.0%	181.53	30.05	31.66
40.0%	181.53	28.95	29.40

* Real rate of interest used in all calculations was four percent.

TABLE 6

Life Insurance Cost Benefit Ratios with Inflation

	Non indexed		Indexed	
Expected Rate of Inflation*	C/B	NC/B	C/B	NC/B
-3.9%	1.7390	.7390	2.3348	1.3348
0.0%	2.4069	1.4069	2.4069	1.4069
5.0%	3.5473	2.5473	2.4994	1.4994
10.0%	4.9317	3.9317	2.5900	1.5900
15.0%	6.4578	5.4578	2.6798	1.6798
25.0%	9.4579	8.4579	2.8556	1.8556
30.0%	10.9252	9.9252	2.9417	1.9417
40.0%	13.5342	12.5342	3.1111	2.1111
R** =	5.62	8.91	1.29	1.50

* Real rate of interest used in all calculations was four percent.

** "R" designates the ratio of cost-benefit ratios under expected inflation rate assumptions of forty percent and zero percent.

Summary and Conclusions

The purpose of this chapter was two-fold: (1) to develop a model capable of measuring the cost of life insurance, and (2) to utilize the model in measuring the effects of inflation and indexation on the cost of life insurance in Brazil.

En route to accomplishing these objectives, a number of interesting by-products emerged. Among the more important were (1) a classification scheme for identifying various degrees of money illusion on the part of the consumer; and (2) the observation that some of the insurance costing procedures serve to reinforce consumer money illusion in one of its forms when employed in an inflationary context.

Finally, a method appropriate for determining the cost of life insurance was introduced. The method was capable of measuring changes in the cost of life insurance incurred by inflation. The model was then applied in determining how the cost of insurance would change under different rates of anticipated inflation. It was demonstrated mathematically that when insurance terms are slow to adjust to the realities of inflation (perhaps due to regulatory constraint), the net cost of insurance would rise in real terms. Actual policy data were then substituted into the mathematical model, and the analytical conclusions were corroborated. In one example it was shown that with an expected inflation rate of forty percent (somewhat below the realized inflation rate of the past three years in Brazil), the net cost per unit of insurance benefit was almost nine times higher than the cost of similar protection under stable prices.

Next, the case of the insured who through various methods attempts to maintain the real value of his insurance was examined. Although there are several approaches the insured could take, investigation was

limited to two of these: purchasing index-linked policies or purchasing additional policies. These two approaches were selected for further investigation for three reasons:

(1) Both approaches are attempts to maintain a real level of protection and are more likely to approximate this goal than other approaches.

(2) The two approaches have been viewed as equivalent by some writers.

(3) Each approach has been said to neutralize the value reduction produced by inflation.

In summary, the investigation resulted in the following findings:

(1) Neither of the two approaches maintains constant a real level of protection against either the peril of premature death or the peril of outliving earning capacity.

(2) Both approaches help in better achieving a desired real level of protection than could be accomplished by the purchase of a single, nonadjusting policy, but at higher premium outlays.

(3) The purchase of an index-linked policy always results in lower expected net present costs, per unit of (real-valued) benefit, than the purchase of additional policies on an annual basis, under positive inflation.

Conceivably, an indexed policy could be designed to maintain constant the real levels of protection (via continuous indexing), but until

such a product is marketed, it can be concluded that a world with stable prices will best promote the objectives of an insured desiring to maintain constant a real level of protection against the perils of dying prematurely or outliving earning capacity.

CHAPTER 3
RATIONAL LIFE INSURANCE PURCHASING AND INFLATION

Survey of the Literature

A number of writers have examined various aspects of rational insurance purchasing, but most of their studies have not dealt explicitly with inflation as an explanatory factor. This is understandable, in part, because inflation rates at the time when many of these studies were published were mild relative to their more recent levels, at least in the United States. Perhaps another reason that insurance purchasing has not been considered in the context of an inflationary environment is that the models used have often been ill-designed for that purpose.[1]

For example, models for rational insurance purchasing are given by Smith (1968), Mossin (1968) and Ehrlich and Becker (1972). Although the models are oriented toward insurance in general, and property insurance in particular, their applicability to the problem of life insurance is straightforward. However, in none of these analyses is

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[1]This, of course, is not to say that the models were ill-contrived. Each model sheds additional light on the theory of optimal life insurance, and all have greatly contributed to the present writer's understanding. The approach set forth in this chapter has benefited from and incorporated components of several of the models.

the dimension of time included (or when included, there is no assumed time preference for wealth — i.e., the rate of interest is set at zero). Since inflation is a phenomenon which, by definition, occurs over time, its significance is effectively precluded from the analyses.

Another set of articles, which are oriented specifically toward life insurance, include the dimension of time. Yaari (1965) and Richard (1977) propose continuous-time models to consider the problem of consumption and portfolio choices when lifetime is uncertain and life insurance is available. The life insurance offered is of an instantaneous term variety where new insurance contracts, which remain in force for an infinitesimally short period of time, are continually bought at each moment in time. While the continuous-time framework is intuitively appealing for analyzing other aspects of their models (such as consumption), the instantaneous term variety of insurance could conceivably result in the prospective consumer spending twenty-four hours of each day at the insurance office applying for new insurance policies, leaving little time for consumption related activities.[2] The loss of the purchasing power of insurance benefits resulting from the inflation that transpires during the interval between premium payment

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[2]If each policy is accompanied by a fixed policy fee (to help offset transaction costs — the usual case) the transaction costs involved could be phenomenal. The closest kind of policy actually offered to the instantaneous term variety is flight insurance, which expires in hours. An ingenious analysis of economic aspects of the purchase of flight insurance is given in Eisner and Strotz (1961).

and insurance settlement is of little or no consequence when the time interval approaches zero. The omission of inflation in their analyses was therefore justified in light of the variety of insurance that was examined.

In a third group of articles life insurance demand is treated in discrete time-period analyses. Hakansson (1969) and Fischer (1973) propose multiperiod models in which the utility functions exhibit constant relative risk aversion. Many of their more specific results turn strongly on their choices of utility functions, and although the models conceivably could have permitted inflation to be included as an explanatory factor, this aspect of the problem was not explicitly considered. Fortune (1973), Jones-Lee (1975) and Klein (1975) have approached the problem of optimal life insurance within the framework of a two-period model (two time-points). Using mean-variance, conditional expected utility, and time-state preference analyses, respectively, each author assumes that the insured either dies immediately after the payment of the initial (and only) life insurance premium, at which time an insurance benefit is paid to the insured's beneficiaries, or that the insured dies (or retires) at the beginning of the second period, at which time he either receives the cash value (savings portion) of the policy or receives nothing, depending on the features of the policy purchased. Under these formulations, inflation's possible impact is restricted to its effect upon the savings that are sought through

the vehicle of life insurance, since in the event of premature death, the ink will not have dried on the policy contract before death occurs. This obviously does not allow time for inflation to affect the level of real protection provided against the peril of premature death. Finally, Razin (1976) presents a two-period (three time-points) model to highlight the effect of lifetime uncertainty on the optimal investment in human capital with and without markets for life insurance. In this case, life insurance is of the single-period term variety, and death occurs either at the end of the first period, or at the end of the second period. This formulation allows for the inclusion of inflation as an explanatory variable, but it is not explicitly introduced into the analysis. Moreover, the particular model he presents treats life insurance in terms of "percentage coverage" instead of absolute coverage, and is thus unable to result in a demand for insurance function (when inflation is allowed to influence the price of insurance coverage) that yields unambiguous predictions with respect to changes in anticipated inflation.

The only theoretical model encountered where inflation was explicitly posited as an explanatory factor for desired levels of life insurance protection[3] is that in Hofflander and Duvall (1966). For this reason, more space will be used here to scrutinize their presentation.

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[3] In this chapter the phrase "life insurance protection" is used to designate protection against the peril of premature death; protection against the peril of outliving earning capacity will be referred to as savings.

Hofflander and Duvall employed the technique of indifference curve analysis in examining the relationship between the amount of (real-valued) life insurance protection purchased and the rate of anticipated inflation. Specifically, they assumed that (1) there were expectations of price level increases in the future time periods; (2) the amounts of other goods and services which can be purchased during the present time period with a given budget remained unchanged, as well as the nominal level of protection; (3) the consumer expects and acts as if the price of real protection has increased; (4) real protection is not an inferior good; and (5) income, employment, and population remain at a given level. From these assumptions their analysis leads them to conclude that expectations of rapidly increasing price levels in the future will lead life insurance consumers to decrease their purchases of life insurance. Their ideas are illustrated below in Figure 4, where I_1 is identified as an indifference curve representing combinations of life insurance protection and other goods and services which give the individual the same level of satisfaction. I_2 is a similar curve but represents a lower level of satisfaction, and the line connecting points A and C is the

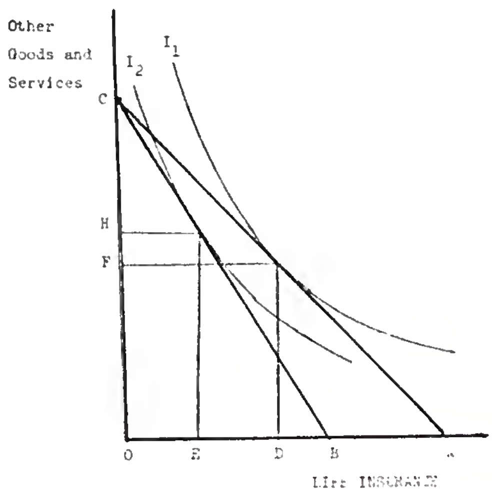

FIGURE 4

individual's budget line, representing attainable combinations of life insurance protection and other goods and services. To reach the highest indifference curve, the individual would purchase OD of insurance and OF of other commodities. Hofflander and Duvall claim that if the individual anticipates a higher rate of inflation in the future, the price of real protection is increased and therefore the budget constraint rotates downward to BD. The new optimal combination will be OE of insurance and OH of other goods and services. Thus, if insurance is a normal good, anticipations of inflation will lead to lowered purchases of real protection.

Neumann (1968) disputes their conclusions on theoretical grounds. He notes that "during an inflationary period not only the price of real protection increases, but also the price of real goods and services goes up... therefore unless the form of the income-consumption curve is known no conclusions can be reached, a priori, as to the direction in which purchases of life insurance would change.[5] He illustrates his claim with Figure 5, at right, where F represents the indifference curve, AA is the initial budget constraint, and R is the optimum combination

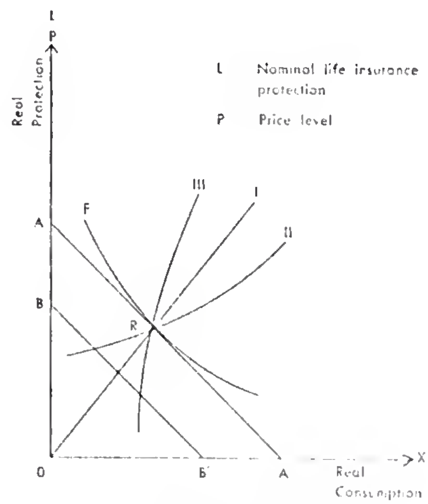

FIGURE 5

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[5]See pp. 629-30 for a more detailed discussion of these points.

point of real protection and real consumption. If prices increase with no change in monetary income, the household suffers losses in both real income and real protection. The budget line shifts toward the origin to BB , and the household suffers a preference loss. The new optimum point will be positioned according to the form of the household's income-consumption curve. Possible shapes of these curves are illustrated by I, II, and III. Therefore, the direction in which life insurance purchases would change depends upon which form of income-consumption curve is appropriate for the family.

In the opinion of this author, Neumann could have made his point simply by recognizing that the Hofflander-Duvall model was a timeless model and hence incapable of furnishing a suitable framework for the analysis of the effect of inflation, which by its very nature occurs over time, upon the demand for protection, which is also received over time. To extend an impotent analysis to its "logical" end, and then to conclude that the implications of the Hofflander-Duvall study are incorrect because they are at variance with those derived by Neumann, who uses a similar timeless model, only compounds the error. Clearly what is required is a model which can take into account time, and its accompanying uncertainty, as well as inflation.

Development of a Theoretical Model

In this section a theoretical model is developed for examining rational life insurance purchasing under inflationary conditions.[6] In accomplishing this objective, the expected utility hypothesis is invoked in a time-state preference framework. The model facilitates a rigorous investigation of the issues of major importance, while laying groundwork for inquiry into other interesting aspects of the life insurance purchase decision.

Background Information

The expected utility model is based on a theorem derived from axioms concerning individual behavior. The theorem on which the model is based dates back to the endeavors of two eighteenth century mathematicians, Daniel Bernoulli and Gabriel Cramer, to resolve the St. Petersburg Paradox.[7] In general terms, the expected utility theorem states that when faced with a set of mutually exclusive actions, each involving its own probability distribution of "outcomes," the individual behaves as if he attaches numbers which are called "utilities"[8] to each

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[6] Recall that in the previous chapter, models were constructed to show inflation's effect upon the cost of life insurance. Nothing was implied about rational purchases.

[7] The St. Petersburg Paradox and the solutions posited by Bernoulli and Cramer are discussed in Levy and Sarnat (1972, Ch. 6).

[8] The use of the term "utilities" is unfortunate in that it gives rise to confusion as to its actual meaning. An excellent clarification of the term is given by Friedman and Savage (1952).

outcome and then chooses that action whose associated probability distribution of outcomes provides maximum expected utility.[9] Subsequent to the publication of the Bernoulli and Cramer solutions to the St. Petersburg Paradox, John von Neumann and Oskar Morgenstern (1947) provided a rigorous axiomatic justification for the use of expected utility to explain choices under conditions of uncertainty. In essence, Neumann and Morgenstern demonstrated that if a decision maker acts in a rational and consistent manner, the expected utility theorem leads to optimal results under conditions of uncertainty.[10]

An extension of the expected utility hypothesis which provides a powerful analytical framework for decision making under uncertainty is the Time-State Preference approach. Evolving from the pioneering works of Arrow (1964), Debreu (1959), and Hirshleifer (1965), this approach assumes that the present values of uncertain future returns depend on the pattern of returns across various states-of-nature, the utility for money in the various states and the likelihood of occurrence of the particular states. Thus, unlike the expected utility hypothesis, the Time-State Preference model explicitly allows for the possibility

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[9]To compute the expected utility of a given probability distribution of outcomes, the utility of each possible outcome is multiplied by the probability of the outcome, and the sum of these products over all possible outcomes is the expected value of utility for the probability distribution of outcomes.

[10]An elegant derivation of the model is in Hernstein and Milnor (1953).

that the value (utility) of income received at a future point in time depends not only upon the length of time between now and then but also the circumstances of the individual when the income is received.[11]

Specification of the Model

The general properties of the model to be employed in the analysis having been stated, the components of the model are now specified.

Discrete-time analysis. The method of discrete-time analysis lends itself readily to the task at hand, as insurance premiums are paid in lump sums at points in time. For purposes of analysis, time is organized into "periods" based upon the natural decision time junctures associated with the periodic incurrence of premiums (to pay or not to pay, and in the case of term insurance extending beyond a single period, to continue or not to continue paying).[12] These decision points are assumed to occur over regular intervals (periods) of time.[13] Events will be assumed to occur at the beginning or the end of each

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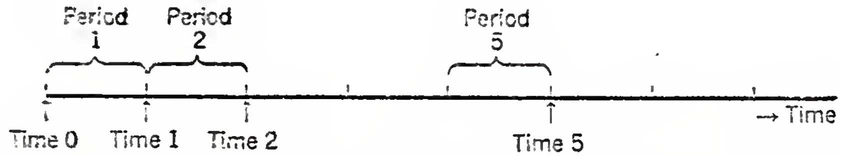
[11] In technical terms, the axiom of uniqueness used in deriving the expected utility hypothesis is relaxed under the Time-State Preference approach. For an excellent discussion of the two approaches and their differences, see Hirshleifer (1970).

[12] Premiums may be incurred on a monthly, quarterly, semi-annual, or an annual basis, and the time periods could be defined to accomodate any of these arrangements. Although it is less common in practice, premiums may be incurred at even lengthier intervals (e.g. single payment life insurance).

[13] See Haley and Schall (1973, Chapter 1) for a more thorough description of discrete-time analysis.

period, unless otherwise indicated. These assumptions are illustrated in Figure 6 below.



The decision maker is now at time 0. Period 1 extends from time 0 to time 1; period 2 extends from time 1 to time 2; etc. Thus, in accordance with the assumptions stated above, events for period 1 occur either at time-point 1 or 2. Throughout this chapter, a subscript will be attached to events of concern to indicate at which point in time they occur.

States of nature. In the Time-State Preference approach, the concept of ''states'' or ''states of nature'' is central. Other characteristics of the model, such as utility functions (or more accurately, preference scaling functions) of individuals, returns on assets, and probabilities, are all based on the definition of states.

> The individual is assumed to have in mind a set of possible states of nature in which each state is a particular sequence of events occurring from the present to a future point in time where the state is defined. In other words, if state s is said to occur at time t, the definition of state s includes a description of relevant events which have happened up to that point. . . only one state can occur at a given point in time (states are mutually exclusive and exhaustive).[14]

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[14] See Haley and Schall (1973, p. 192).

In the present discussion, two time-states will be explicitly considered at each point in time.[15] Uncertainty of lifetime is the primary concern here, and other uncertainties with which a consumer must normally cope are ignored. Thus, the uncertainty of future earnings, for example, will enter the model only insofar as the flow of earnings stops when the consumer dies.

A multiperiod model will be used to examine rational life insurance purchasing under inflation. It is assumed that the individual will die sooner or later, where "sooner" will correspond to death during the first period, and "later" will correspond to death during a subsequent period. There is a maximum number of periods, T, for which the individual can live. If he is alive at time T, he will be dead at T + 1.[16] The individual will be referred to as the "breadwinner," denoting primary responsibility for the financial welfare of his dependents. If the breadwinner dies during period 1, his heirs will not receive from the estate that value which would have accrued to them had the breadwinner survived that (and perhaps subsequent) period(s). It is this potential loss against which the breadwinner may desire protection for his heirs.

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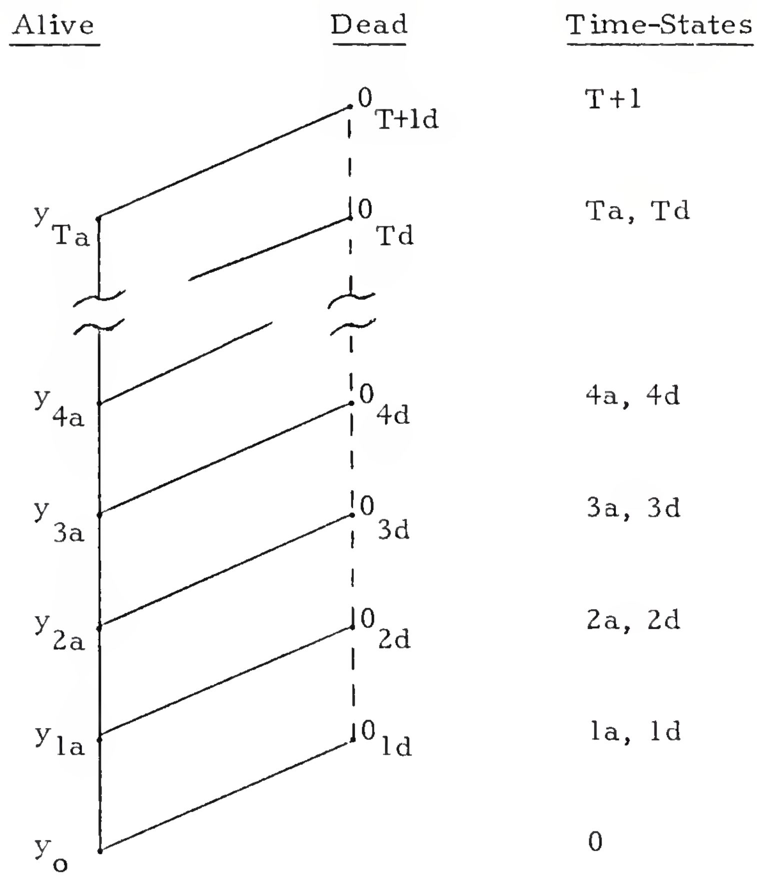
[15]These states need not always be distinct from each other.

[16]The letter T can be set to represent any length of time necessary (say 200 years) to ensure that the assumption approximates reality for all practical purposes.

The possible time path sequences considered in the model are pictured in Figure 7.[17]



Sequence (0, 1d, 2d, 3d, 4d, ...) will be denoted State T1;
Sequence (0, 1a, 2d, 3d, 4d, ...) will be denoted State T2;
Sequence (0, 1a, 2a, 3d, 4d, ...) will be denoted State T3;
and so on.

FIGURE 7: Tree of Time-State Sequences[18]

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[17] The possibilities of a future incarnation or resurrection are not considered in this model, although the time-state preference framework may be sufficiently flexible to incorporate such events.

[18] The symbols preceding the time-state subscripts denote the real value of disposable income received in that time-state deriving from human capital (see pp. 89-91).

Probabilities. Probabilities are specified in terms of the likelihood of a particular state occurring at a given point in time. Because states are mutually exclusive and exhaustive, the sum of probabilities over all possible state sequences is equal to unity.

In the present model, the probabilities associated with the occurrence of each state are dependent upon the likelihood of breadwinner mortality at a given point in time. The notation π_{1d} will denote the probability of death occurring to the breadwinner during the first period, and π_{1a} will denote the probability of surviving the first period (such that the breadwinner is alive at the second decision point, time 1). Since it is assumed that the individual will either die in period one or will not die in period one, $(1-\pi_{1d})$ will be equal to π_{1a}. Thus, the probability of sequence (0, 1d, 2d, . . .) is π_{1d}, while the probabilities of all other sequences of states occurring, when summed over all possible sequences, will equal $(1-\pi_{1d})$, or simply π_{1a}.

Wealth status. The resources of the breadwinner consist of non-human (physical) capital and human capital, each of which can generate future income streams. Physical capital is inheritable, and its value will be considered independent of breadwinner lifetime. The present value of the income generated by the physical capital with which the individual is endowed at time 0, where uncertainty is limited to length

of life, will be equal to the value of the assets, and will be denoted A_o.[19]

Human wealth. The valuation procedure applicable for human wealth is somewhat more complicated, because of the uncertainty that surrounds lifetime. In this model, the potential income from human capital (which may consist of salary, wages, pensions, unemployment compensation, and so forth) may possess any pattern over time, but it is assumed to be known in advance and to terminate upon death of the breadwinner.[20] It is further assumed that the real rate of return on human capital is independent of the rate of inflation.[21] The rate of interest is presumed to be known but may have any pattern over time.

If the individual is alive at the decision point t, he will be paid the (after-tax) installment pertaining to the period t + 1 at the beginning of that period (i.e., at time t). The real value of this installment will be denoted y_t, where $y_t \geq 0$. If he is not alive at that point, no income will be received. Under these conditions, the present value of the individual's potential disposable income stream deriving from human

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[19] It is assumed that the nominal return on physical capital adjusts to fully compensate for any inflation that may transpire.

[20] The word "potential" is used here because the income is contingent upon the survival of the individual, whose lifetime is uncertain.

[21] This assumption is not as strong as it may appear; in Brazil, wages are linked to a price level index to help neutralize inflation's impact on real values.

capital, at the present decision point (time 0), is given by the formula:[22]

$$Y_o^p = y_o + \frac{y_1 J_1}{R_1 J_1} + \frac{y_2 J_1 J_2}{R_1 R_2 J_1 J_2} + \ldots + \frac{y_T J_1 \ldots J_T}{R_1 \ldots R_T J_1 \ldots J_T}, \quad (19)$$

which simplifies to

$$Y_o^p = y_o + \frac{y_1}{R_1} + \frac{y_2}{R_1 R_2} + \ldots + \frac{y_T}{R_1 \ldots R_T}, \quad (20)$$

where

$R_t = (1 + r_t)$;

$J_t = (1 + j_t)$;

r_t = the real risk-free (after-tax) rate of interest expected to prevail during period t (i.e., the time interval between time t-1 and time t); and

j_t = the expected rate of inflation associated with period t.

Yaari (1965), Fischer (1973), and Richard (1977) have shown that when insurance is available, there is an amount of human capital, independent of risky market opportunities and preferences, which is the present certainty-equivalent value for the individual's future (human capital) earnings stream which is assumed to be sure if the individual is alive. This human capital term is calculated by discounting the

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[22]The nominal level of the after-tax installment in any future period t, $y_t \prod_{k=1}^{t} J_k$, adjusts so that it exactly compensates for inflation's effect upon the corresponding discount rate.

future earnings stream until the maximum time of death at a discount rate equal to the product of (1) the risk-free (after-tax) rate of interest plus one, $I_t = 1+i_t = (1+r_t)(1+j_t)$, and (2) the insurance rate applicable at that time plus one, $X_t = 1+x_t$, where

$$x_t = \frac{P_{t-1}}{\dfrac{INS_t}{I_t} - P_{t-1}}, \qquad (21)$$

and P_{t-1} is the premium payable at time t-1 for a given nominal amount of insurance to remain in force throughout the t^{th} period, INS_t. (The actual amount of insurance in force, if any, is immaterial if the insurance rate per unit of insurance in force is invariant with regard to the number of units purchased.)[23] Thus, if the breadwinner is alive at time t, his present certainty-equivalent value (at time t) of disposable income to be received is given by

$${}_tY_{ta}^{ce} = y_t + \frac{y_{t+1}J_{t+1}}{I_{t+1}X_{t+1}} + \frac{y_{t+2}J_{t+1}J_{t+2}}{I_{t+1}I_{t+2}X_{t+1}X_{t+2}} + \ldots \qquad (22)$$

which simplifies to

$${}_tY_{ta}^{ce} = y_t + \frac{y_{t+1}}{R_{t+1}X_{t+1}} + \frac{y_{t+2}}{R_{t+1}R_{t+2}X_{t+1}X_{t+2}} + \ldots \qquad (23)$$

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[23]Note that the determination of the present certainty-equivalent value of income does not require that any insurance be actually purchased.

where the subscript preceding the Y term indicates the time back to which income is discounted, and the subscript appearing after Y, ta, indicates individual is alive at point t. The superscript ce indicates certainty-equivalent.

Since the individual is permitted to choose whether or not to (continue to) insure at each decision point, and how much to insure, his concern at any point in time is in providing coverage up until the next decision point. In the model developed here, the certainty-equivalent value that will be of particular importance is ${}_1Y_{1a}^{ce}$ since this corresponds to the amount of income forfeited if death occurs during the first period.[24] This value, expressed in currency units valued at time 1, can be discounted back to the present (time 0) to facilitate comparison with the present value of physical capital. The present certainty-equivalent value (at time 0) of all future[25] returns on human capital, conditional upon breadwinner survival of the first period, is given by[26]

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[24]If the individual survives the first period, he will have another chance to adjust his insurance levels for future periods.

[25]Recall that the breadwinner receives y_o immediately and with certainty.

[26]Because this certainty-equivalent value is conditional upon survival of the first period, the discount factors for y_t do not reflect the insurance factor for first period income, X_1.

$$ {}_{o}Y^{ce}_{1a} = \frac{{}_{1}Y^{ce}_{1a}}{R_1} = \frac{y_1}{R_1} + \frac{y_2}{R_1 R_2 X_2} + \frac{y_3}{R_1 R_2 R_3 X_2 X_3} + \ldots \qquad (24) $$

This particular value will figure importantly in the model shortly to be presented. All of the possible sequences of return on human capital associated with states T2, T3, . . . , TT+1 can be collapsed into a single present certainty-equivalent value, ${}_{o}Y^{ce}_{1a}$, which will be added to endowed physical wealth, A_o, and first period income, y_o, conditional upon breadwinner survival of the first period. The probability that the individual's endowed wealth, W^e_a, will result in this sum is π_{1a}. If the individual does not survive the first period, his endowed wealth, W^e_d, will include only the endowed physical wealth and the first period income.[27] The probability of his endowed wealth amounting to this sum is π_{1d}. These are the two conditional outcomes with which the breadwinner is concerned and for which he must make provision at decision point 0.

Insurance. It is assumed that claims to consumption provided by wealth in states T2, T3, . . . , TT+1 , W^e_a, can be traded for claims to consumption provided by wealth in state T1, W^e_d, at the fixed rate[28]

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[27]The model could be easily modified to allow for additional future claims derived from social insurance programs by adding a term to that state's wealth. This extension is not undertaken here, although the implications of the existence and expansion of such programs will be discussed later in the chapter.

[28]The superscript "e" indicates "endowed" wealth.

$$\frac{-dW_a}{dW_d} = x_1, \tag{25}$$

where x_1, defined as before, can be called the insurance rate applicable for period one.

The kind of insurance incorporated in the model is term insurance. Single-period term insurance was selected for analysis for three reasons: (1) there is no loss of generality in using term insurance since all available life insurance is a linear combination of one period (year) term insurance and a savings plan of some sort (Richard, 1977); (2) there are persuasive arguments that other types of insurance (i.e., investment insurance) may be suboptimal (see, for example, Aponte and Denenberg, 1968, and Klein, 1975); and (3) the effect of anticipated inflation on saving through life insurance has already been examined (Neumann, 1969, Fortune, 1972).[29]

Preference scaling functions for contingent wealth. In addition to being endowed with consumption claims (wealth) over different states, and having opportunities for transforming his endowed bundle

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[29]The economics of this problem can be summarized with a statement by George Stigler (1966, p. 57) made in another context, by substituting phrases relating to life insurance, where appropriate: In addition to the yield on life insurance savings, one can explain the participation in a life-insurance "forced-saving" plan by introducing another item of preference: a desire of people to protect themselves against a future lack of will power. If we stopped the analysis with this explanation, we would turn utility into a tautology: a reason, we would be saying, can always be found for whatever we observe a man

of claims into alternative combinations (via insurance), the representative individual (breadwinner) has preference relations ordering the combinations he could possibly hold. The breadwinner may be thought of as deriving a positive satisfaction (utility) from the consumption provided by his claims (wealth) while alive, and to derive positive utility during the period that he is alive for the provision he has made for those who survive him in the event of death.

At the present time his preference toward wealth in the two states for which he must provide at decision point 0 are represented by two preference scaling functions, v_a and v_d. Each function is assumed to be continuous and differentiable at all points, and unique up to a linear transformation.[30] His decision rule is to maximize the expected value of a preference index, U, which is defined as the sum of the values of the two preference scaling functions, weighted in

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to do.

In order to preserve the predictive power of the utility theory, we must continue our insurance "forced-saving" analysis as follows. The foregone cost of putting money in an investment life insurance policy is the additional interest one could earn by putting the same money in a savings account. If interest rates on savings accounts rise while the yields on life insurance savings remain fixed, the cost of buying protection against a loss of will power rises and less of it ought to be bought: relatively more savings will go into savings accounts, relatively less savings into life insurance.

[30]Hirshleifer (1965, Appendix) has given a procedure for a common scaling of preference functions that are conditional upon states consistent with the Neumann-Morgenstern postulates.

accordance with the probabilities of occurrence of their associated states. The maximization of this index will be dubbed, in the tradition of economic literature, ''maximizing expected utility.''

Before the complete model is given, a convenient summary of notation, along with certain identities, and some useful derivatives, are listed in the section that follows.

Notation, Identities, Derivatives

$R_t = 1+r_t$ represents the real discount factor applicable to risk-free (after-tax) cash flows for the time interval t-1 to t, or period t.

$J_t = 1+j_t$ represents the component of the nominal discount factor deriving from expected inflation at rate j_t during period t.

$I_t = 1+i_t = R_t J_t$ represents the discount factor applicable for period t.

π_{1d} is the probability of dying during the first period.

$\pi_{1a} = 1-\pi_{1d}$ is the probability of surviving the first period.

INS_t is the nominal amount of insurance paid to beneficiaries at point t if breadwinner dies during period t; alternatively, it may be considered the amount of insurance in force during period t.

P_t is the premium paid at point t for insurance coverage (of INS_{t+1}) during the subsequent period.

$X_t = 1+x_t = 1+P_{t-1}/(INS_t/I_t - P_{t-1}) = (INS_t/I_t)/(INS_t/I_t - P_{t-1})$ denotes the insurance portion of the discount factor applied to cash flows contingent upon breadwinner survival of period t, equal to the insurance rate for period t income plus one.

y_t denotes the real-valued income to be received at time-point t if the breadwinner is alive at that point.

A_o is the value of endowed physical wealth at time 0.

${}_tY_{ta}^{ce}$ represents the present certainty-equivalent value at time t of disposable income to be received, where the subscript preceding the Y term indicates the point in time back to which income is discounted, and the dual subscript appearing after Y, ta, indicates the individual is alive at point t.

v_a, v_d are the preference scaling functions for wealth conditional on first period survival and mortality, respectively.

$W_a^e = A_o + y_o + {}_oY_{1a}^{ce}$ represents the present value of endowed human and nonhuman wealth at time 0 assuming that the breadwinner survives the first period.

$W_d^e = A_0 + y_o$ represents the present value of endowed human and nonhuman wealth at time 0 assuming breadwinner mortality during the first period.

$W_a = A_o + y_o + {}_oY_{1a}^{ce} - P_o$ is the present value of human and nonhuman wealth at time 0 adjusted for insurance premium outlays at time 0 conditional upon the breadwinner survival of the first period.

$W_d = A_o + y_o + INS_1/R_1J_1 - P_o$ is the present value of human and nonhuman wealth at time zero adjusted for insurance indemnification received at time 1 and insurance premium outlays at time 0 conditional upon breadwinner mortality during the first period.

$P_o = W_a^e - W_a = x_1 (W_d - W_d^e) = x_1S_1$ where x_1 is the price per unit of (present valued) net insurance in force during period one, and $S_1 = W_d - W_d^e = INS_1/I_1 - P_o$ is the present value of net insurance in force during period one.

$x_1 = P_o/(INS_1/I_1 - P_o) = (W_a^e - W_a)/(W_d - W_d^e)$;

$dX_t/dJ_t > 0$ for all t; $dX_1/dJ_t = 0$ for all $t \geq 2$; $dX_t/dJ_t = dx_t/dJ_t$;

$dX_t/dJ_1 = 0$ for all $t \geq 2$;

$$dA_o/dJ_t = 0 \text{ for all } t; \quad dR_t/dJ_t = 0 \text{ for all } t;$$[31]

$$dW_d^e/dJ_t = 0 \text{ for all } t; \quad d(y_t/R_1 \ldots R_t)/dJ_t = 0 \text{ for all } t;$$

$$d_oY_{1a}^{ce}/dX_1 = 0; \quad d_oY_{1a}^{ce}/dX_t < 0 \text{ for all } t \geqq 2;$$

$$d_oY_{1a}^{ce}/dJ_1 = d_oY_{1a}^{ce}/dX_1 \cdot dX_1/dJ_1 = 0; \quad dW_a^e/dJ_1 = 0;$$

$$d_oY_{1a}^{ce}/dJ_t < 0 \text{ for all } t \geqq 2; \quad dW_a^e/dJ_t < 0 \text{ for all } t \geqq 2.$$

These derivatives will be of use in the comparative statics formulas to be derived in the next section. Having specified the formal relations of importance, the stage is set for investigating rational life insurance purchasing and anticipated inflation.

Optimal Life Insurance

To determine the optimal distribution of consumption claims across states, the expected utility of wealth prospects is maximized by maximizing the objective function

$$U = \pi_{1a} v_a(W_a) + \pi_{1d} v_d(W_d) \tag{26}$$

subject to the opportunities for wealth transformation across states available through the trading conditions

$$W_a^e - W_a - x_1(W_d - W_d^e) = 0. \tag{27}$$

This problem can be handled in the usual way by setting up the Lagrangean function and taking its derivatives with respect to the

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[31]This reflects the assumption made throughout the study and not a derivation *per se*.

variables W_a, W_d and @ (where the symbol @ represents some as yet undetermined multiplier) as follows:

$$\underset{W_a, W_d, @}{\text{Maximize}} \quad L = \pi_{1a} v_a(W_a) + \pi_{1d} v_d(W_d) - @\left[W_a^e - W_a - x_1(W_d - W_d^e)\right] \tag{28}$$

$$= \pi_{1a} v_a(W_a) + \pi_{1d} v_d(W_d) - @\left[W_d^e + {}_oY_{1a}^{ce} - W_a - x_1(W_d - W_d^e)\right]$$

$$\frac{\delta L}{\delta W_a} = \pi_{1a} v_a' (W_a) + @ = 0 \tag{29}$$

$$\frac{\delta L}{\delta W_d} = \pi_{1d} v_a' (W_d) + @x_1 = 0 \tag{30}$$

$$\frac{\delta L}{\delta @} = W_d^e + {}_oY_{1a}^{ce} - W_a - x_1(W_d - W_d^e) = 0 \tag{31}$$

The resulting first-order optimality conditions are thus:

$$x_1 \pi_{1a} v_a'(W_a) - \pi_{1d} v_d'(W_d) = 0 \tag{32}$$

$$W_d^e + {}_oY_{1a}^{ce} - W_a - x_1(W_d - W_d^e) = 0 \tag{33}$$

The values which fulfill these conditions will maximize expected utility if the determinant of the bordered Hessian, $|\bar{H}|$, is greater than zero. Accordingly, the second-order condition for optimality is given by

$$|\bar{H}| = \begin{vmatrix} 0 & -1 & -x_1 \\ -1 & \pi_{1a} v_a''(W_a) & 0 \\ -x_1 & 0 & \pi_{1d} v_d''(W_d) \end{vmatrix} \overset{?}{>} 0 \tag{34}$$

$$= -x_1^2 \pi_{1a} v_a''(W_a) - \pi_{1d} v_d''(W_d)$$

provided that $v_a', v_d' > 0$ and $v_a'', v_d'' < 0$, which will be the case if, other things equal, the individual will prefer more wealth to less wealth in

any given state, while exhibiting decreasing marginal utility of wealth in each state. Thus, the optimum amount of net insurance in force during period one, in present value terms, S_1, is attained when

$$S_1 = \frac{P_o \pi_{1a} v'_a(W_a)}{\pi_{1d} v'_d(W_d)} . \tag{35}$$

Rational Life Insurance Purchasing and Inflation

The manner in which the optimal amount of life insurance can be expected to change when inflation is anticipated can be determined by totally differentiating the two first-order conditions with respect to the rate of inflation anticipated in a given period. If inflation at rate j_1 is anticipated during the first period, but expected to vanish thereafter, its effect on rational insurance purchasing is given by totally differentiating (32) and (33) with respect to J_1.

$$\frac{dx_1}{dJ_1} \pi_{1a} v'_a(W_a) + x_1 \pi_{1a} v''_a(W_a) \frac{dW_a}{dJ_1} - \pi_{1d} v''_d(W_d) \frac{dW_d}{dJ_1} = 0 \tag{36}$$

$$\frac{d\, {}_oY^{ce}_{1a}}{dJ_1} - \frac{dW_a}{dJ_1} - \frac{dx_1}{dJ_1}(W_d - W^e_d) - x_1 \frac{dW_d}{dJ_1} = 0 \tag{37}$$

which can be arranged as

$$\frac{dW_a}{dJ_1} = \frac{d\, {}_oY^{ce}_{1a}}{dJ_1} - \frac{dx_1}{dJ_1}(W_d - W^e_d) - x_1 \frac{dW_d}{dJ_1} . \tag{38}$$

Substituting (38) into (36) yields

$$\frac{dx_1}{dJ_1}\pi_{1a}v_a'(W_a) + x_1\pi_{1a}v_a''(W_a)\left[\frac{d\,_oY_{1a}^{ce}}{dJ_1} - \frac{dx_1}{dJ_1}(W_d - W_d^e)\right] - x_1\frac{dW_d}{dJ_1}$$

$$- \pi_{1d}v_d''(W_d)\frac{dW_d}{dJ_1} = 0 \; . \tag{39}$$

Solving (39) for dW_d/dJ_1 gives

$$\frac{dW_d}{dJ_1} = \frac{\frac{dx_1}{dJ_1}\pi_{1a}v_a'(W_a) + x_1\pi_{1a}v_a''(W_a)\left[\frac{d\,_oY_{1a}^{ce}}{dJ_1} - \frac{dx_1}{dJ_1}(W_d - W_d^e)\right]}{x_1^2\pi_{1a}v_a''(W_a) + \pi_{1d}v_d''(W_d)} \overset{?}{<} 0, \tag{40}$$

where the sign of the inequality is resolved upon determining whether the sum of $(W_d - W_d^e)$ is negative or positive. The typical case for anyone who purchases any insurance is for $(W_d - W_d^e) \geqq 0$. This observation, combined with the information given in the section on definitions and derivatives, leads to the unambiguous conclusion that $dW_d/dJ_1 < 0$.

Noting that

$$\frac{dS_1}{dJ_1} = \frac{dW_d}{dJ_1} + \frac{dW_d^e}{dJ_1} = \frac{dW_d}{dJ_1} < 0,$$

which means that an increase in anticipated inflation for the first period will unambiguously have a negative effect upon the net purchases of life insurance protection (measured in present currency units).

Next, the effect of inflation that is anticipated beyond the first period upon rational life insurance purchasing is examined. To determine its effect, the first-order optimality equations (32) and (33) are totally differentiated with respect to future period expected inflation, J_t, where $t \geqq 2$:

$$\frac{dx_1}{dJ_t}\pi_{1a}v_a'(W_a) + x_1\pi_{1a}v_a''(W_a)\frac{dW_a}{dJ_t} - \pi_{1d}v_d''(W_d)\frac{dW_d}{dJ_t} = 0. \tag{41}$$

Because $dx_1/dJ_t = 0$ for $t \geqq 2$, the above expression simplifies to

$$x_1\pi_{1a}v_a''(W_a)\frac{dW_a}{dJ_t} - \pi_{1d}v_d''(W_d)\frac{dW_d}{dJ_t} = 0.$$

$$\frac{dW_d^e}{dJ_t} + \frac{d_oY_{1a}^{ce}}{dJ_t} - \frac{dW_a}{dJ_t} - \frac{dx_1}{dJ_t}(W_d - W_d^e) - x_1\frac{dW_d}{dJ_t} + x_1\frac{dW_d^e}{dJ_t} = 0 \tag{42}$$

The first, fourth and sixth terms of the above equation are all zero, so that the equation may be simplified and rearranged to

$$\frac{dW_a}{dJ_t} = \frac{d_oY_{1a}^{ce}}{dJ_t} - x_1\frac{dW_d}{dJ_t}. \tag{43}$$

Substituting this result into (41) gives

$$x_1\pi_{1a}v_a''(W_a)\frac{d_oY_{1a}^{ce}}{dJ_t} - x_1\frac{dW_d}{dJ_t} - \pi_{1d}v_d''(W_d)\frac{dW_d}{dJ_t} = 0. \tag{44}$$

Solving (44) for dW_d/dJ_t results in

$$\frac{dW_d}{dJ_t} = \frac{x_1\pi_{1a}v_a''(W_a)(d_oY_{1a}^{ce}/dJ_t)}{x_1^2\pi_{1a}v_a''(W_a) + \pi_{1d}v_d''(W_d)} < 0, \Longrightarrow \frac{dS_1}{dJ_t} < 0, \tag{45}$$

so that an increase in any J_t, $t \geqq 2$, will result in a decrease in (net) purchases of insurance protection (valued in present currency units) during the first period.

Of particular interest is the fact that rational insurance purchasing will decline regardless of when rising inflation is anticipated to occur. If inflation is expected to accelerate during the first period, the negative impact on life insurance purchasing will be due

entirely to inflation's direct effect upon the cost of real protection during that period. But if inflation is anticipated to increase in subsequent periods, the effect will be indirect; that is, the entire effect will be due to the higher discount rates applied to future real income flows (in determining the present certainty-equivalent value of income) resulting only from increased insurance rates. Thus, even if nominal income fully adjusts to the inflation-produced value erosion, human wealth is lessened by inflation when insurance rates are fixed or slow to adjust.[32] Of course, the converse of these propositions are also true: expectations of declining inflation will lead to increased purchases of net insurance (in present value units), as well as a higher valuation upon human capital.

Several additional theoretical relationships could be derived from the optimality conditions. One which is of particular importance in virtually every empirical study, and the only other one dealt with here, is the relationship between real income and insurance purchasing. Following the usual procedure, equations (32) and (33) are totally differentiated with respect to y_t, yielding

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[32] Note that in a world of certainty, the insurance policy conditions could conceivably and safely incorporate higher interest earnings associated with inflation, leading to a reduction in nominal premium charges were it not for unaccomodating regulators. In a world of uncertainty, however, where the pattern of future inflation is unknown, insurers could never fully adjust to the expected inflation rates because deviations from the expected rates could place the insurer in jeopardy of insolvency. Thus, unless any excess returns resulting from the margin of safety inherent in the insurance rates are returned to policyowners (perhaps through a dividend or subsequent premium reduction), a loss is suffered by the insured. Provisions for returning excess profits to the insured are not in existence in Brazil at the present time.

$$x_1\pi_{1a}v_a''(W_a)\frac{dW_a}{dy_t} \;-\; \pi_{1d}v_d''(W_d)\frac{dW_d}{dy_t} \;=\; 0; \tag{46}$$

$$\frac{dW_d^e}{dy_t} + \frac{d_oY_{1a}^{ce}}{dy_t} - \frac{dW_a}{dy_t} - x_1\frac{dW_d}{dy_t} + x_1\frac{dW_d^e}{dy_t} \;=\; 0. \tag{47}$$

Substituting the value implicit in (47) for dW_a/dy_t into (46) and solving for dW_d/dy_t leaves

$$\frac{dW_d}{dy_t} = \frac{x_1\pi_{1a}v_a''(W_a)\left[\frac{dW_d^e}{dy_t} + \frac{d_oY_{1a}^{ce}}{dy_t} + x_1\frac{dW_d^e}{dy_1}\right]}{x_1^2\pi_{1a}v_a''(W_a) + \pi_{1d}v_d''(W_d)} > 0 \quad \text{for any } t. \tag{48}$$

However, the relationship between (net) insurance purchases and an increase in real income is not always unambiguous. For $t > 0$, the relationship is unambiguous since $dS_1/dy_t = dW_d/dy_t - dW_d^e/dy_t$, which simplifies to $dW_d/dy_t > 0$, so that an increase in expected real future income leads to higher insurance purchases at present. This is reasonable, since a greater value will be potentially forfeitable in the event of death if real income is expected to rise. On the other hand, the relationship is ambiguous for $t = 0$. In this case, the equation is tautological, since

$$\frac{dS_1}{dy_o} = \frac{dW_d}{dy_o} - \frac{dW_d^e}{dy_o} = \frac{d(A_o + y_o + INS_1/R_1J_1 - P_o)}{dy_o} - \frac{d(A_o + y_o)}{dy_o} . \tag{49}$$

Because $dA_o/dy_o = 0$, and because $S_1 = INS_1/R_1J_1 - P_o$ by definition, the equation reduces to

$$\frac{dS_1}{dy_o} = 0 + 1 + \frac{dS_1}{dy_o} - 0 - 1 = \frac{dS_1}{dy_o} \overset{?}{\gtrless} 0 .$$

This result is intuitive: an equal increase in consumption claims in both states leaves unchanged the absolute amount of value forfeitable

upon premature death, so that any changes in the level of insurance purchases will be due to differences in absolute risk aversion when endowed wealth increases by equal amounts in both states.[33]

Of course, if an increase in real wage income at time zero engenders expectations of future increases in real wage income and the positive effect of increased wage earnings in several or all future time periods swamps the undecided effect of the increase at time zero, additional insurance will be purchased.

In summary, the analysis leads to two propositions which will be subjected to empirical verification in the next chapter. Before the propositions are stated, it must be remembered that because of the manner in which the policy values of term life insurance are annually adjusted in Brazil,[34] all of the algebraic derivations of this chapter are equally valid for indexed, as well as nonindexed policies.

Proposition 1: An increase in expected inflation will lead to a decline in the amount of (net) insurance purchases (valued in present currency units); conversely, a decrease in expected inflation will lead to an increase in the amount of (net) insurance purchases (valued in present currency units).

Proposition 2: An increase in expected real wage income will lead to increased purchases of (net) insurance (valued in present currency units); conversely, a decrease in expected real wage income will lead to decreased purchases of (net) insurance (valued in present currency units).

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[33]Arrow (1964) and Pratt (1964) derive measures of relative and absolute risk aversion and theorize about the shape of these functions.

[34]Recall from Chapter 2 that the cost per unit of protection was identical for indexed and nonindexed term policies, except for possible nuisance renewal costs. The cost advantage of indexed investment life policies over nonindexed investment life policies would not enter into the model unless term insurance (or group term insurance) were unavailable to the consumer of life insurance.

A Methodological Note

The theoretical model developed in this chapter has incorporated several features which render it attractive for carrying out the analysis, as well as some limitations.

Virtues

Among the special features of the model which enhance its suitability for analyzing rational life insurance purchasing and inflation, and which in combination make the methodology proposed here unique, are those contained in the partial list below.

(1) Discrete-time analysis lends itself admirably to the task at hand since income, insurance indemnification, and insurance premiums are all events which generally occur at discrete time points. Moreover, although insurance contracts often are scheduled to expire well into the future, the periodic nature of premium payments effectively presents the insured with successive decision time-points. The discrete-time framework also readily accomodates the inclusion of inflation as an argument.

(2) Instead of the common two-period model, the multiperiod design adopted here allows for the representative individual to exhibit concern not only for the possible forfeiture of a single period of income due to premature death, but for the possible loss of income relating to an entire (normal) lifetime. This construction promotes insurance purchases in large quantities consistent with lifetime contingencies,

while continuing to present future decision points. The long length of time also allows for the application of realistic death probabilities.

(3) The insurance discussed here, following the lead of Ehrlich and Becker (1972), is net insurance, or coverage minus premiums. This definition, in addition to its intuitive appeal, lends itself readily to Time-State Preference analysis and also enables unambiguous propositions relating insurance and inflation to be derived. It also lends itself to empirical analysis better than insurance defined in terms of "percentage coverage," as in Smith's (1968), Mossin's (1968), and Razin's (1976) papers.

(4) Time-State Preference theory is a powerful analytical tool which is especially well designed for problems of choice under uncertainty. Hirshleifer (1965, 1966, 1970) and Robichek (1969) have advocated, but never formally implemented, its use in analyzing insurance purchasing decisions. The states of life or death at a given point in time are obvious in nature, and easily adopted into the model.

(5) Unlike most previous studies using Time-State Preference theory, the intuitively uncomfortable assumption of a unique preference scaling function (Klein, 1975, Ehrlich and Becker, 1972), regardless of time or state, was not resorted to in quest of convenience or mathematical tractability. Rather, the more reasonable assumption was employed that the individual's utility for wealth is dependent upon the state of nature (in particular, his future life-death status).

(6) Although the Time-State Preference framework requires the existence of certainty, given the occurrence of a state, this posed no hardship upon the analysis here. In the event of mortality the amount of the death benefit is known with certainty, since it is established in the contract beforehand. Coupling that with a known pattern of interest rates, there is no uncertainty about the value of the payment. In the event of first period survival, there was a convenient present certainty-equivalent value of income to apply in that state.

(7) Finally, only the most general restrictions were placed on the shapes of the utility functions, restrictions which are supported by a large body of evidence (Levy and Sarnat, 1972). Thus, the implications should be of greater generality.

Limitations

In achieving virtues of simplicity, a tradeoff has resulted: some rather restrictive assumptions were needed. Furthermore, the general form of the analysis has several limitations. These unpleasantries are discussed below.

(1) Perhaps the most severe restriction placed on the model was the limitation of uncertainty to the duration of life. The model could have permitted uncertainty with regard to other of its elements, such as income, interest rate and inflation rate patterns, as well as returns on physical wealth. Imperfect information (Rothschild and Stiglitz, 1976) with regard to death probabilities could also have been included

in the model. Stochastic differentiation techniques or an expansion in the number of states considered at each time point are techniques available for implementation of such uncertainties.

(2) Death benefits were assumed to be payable at the end of the period during which death occurs. A more realistic assumption would have been to have death benefits payable upon death. Coupled with interest rate uncertainty, the additional uncertainty associated with intraperiod timing of death and the receipt of the death benefit would exacerbate the uncertainty surrounding the value of the death benefit. Methodologies are being developed to accomodate the uncertainty deriving from both the timing and the amount of uncertain future cash flows (Perrakis, 1975), and may perhaps be incorporated in a future study.

(3) The complexities of tradeoffs between labor and leisure (Ryder, Stafford, and Stephan, 1976) and labor and education (Razin, 1976), and their relationship to the life insurance decision were ignored, although they have been studied elsewhere.

(4) Consumption and investment earnings take place almost continuously over time, although income, insurance premiums and payments generally occur in periodic discrete cash flows. By proposing a discrete-time framework, some elements of the model were forced into distorted abstractions. The use of Stieltjes-Lebesques integrals may have captured the appeal of both discrete and continuous-time analyses.

(5) The analysis performed here was only partial equilibrium analysis.

(6) The model abstracted from the problems of insurability and policy renewability, and social security claims payable in the event of breadwinner mortality were not taken into account. In the latter case, however, additional endowed wealth in any state can be expected to increase the demand for adjusted claims in all states; if insurance is the adjustment mechanism, less of it will be purchased when social security augments claims to consumption only in state "breadwinner dies."

(7) Dividends payable in participating life insurance policies were not considered here. That is because there are no participating policies available in Brazil.

In conclusion, several problems are present in the methodology presented. However, economic technology capable of alleviating most of these problems is already in existence, and could be summoned, but at a price of simplicity.

CHAPTER 4
TESTS OF THE HYPOTHESES: RESEARCH RESULTS

The purpose of this chapter is to examine empirically the theoretical relationships, specified earlier as propositions, for the Brazilian case. Following the tests, the major research results of this dissertation will be summarized.

The Model

A time-series multivariate regression model is used here to relate explanatory variables such as inflationary expectations and income to life insurance purchases. The model will be estimated using ordinary least squares.

The regression analysis will be applied to two separate periods: (1) the preindexing period prior to 1968; and (2) the postindexing period beginning in 1968.[1] The preindexing period examined begins in 1951. This is because a severe abnormality was encountered in the measured life insurance premiums collected during 1950 which could not be explained by the model presented here. From 1949 to 1950, life insurance premiums paid in Brazil rose approximately 45 percent in

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[1]Indexing in Brazil received its main implementation in 1964 (Baer and Beckerman, 1976) but it was not until November of 1966 that indexing was authorized for life insurance contracts, and not until a year later that the first company, Piratininga, marketed an index-linked policy.

real values. Subsequently, from 1950 to 1951, they fell an even greater percentage in real terms. Inflation and income, separately or in concert, were unable to explain such aberrations. The cause of the phenomenon was related to a tax provision which permitted full deductibility from taxable income any amount of life insurance premiums incurred. The Brazilian life insurance industry responded quickly to this opportunity by creating single year endowment policies. To minimize tax liability, a consumer could purchase an endowment policy of large denomination and deduct his premium expenditure from taxable income. At the end of the year, if the insured remained alive, he would surrender the policy in return for its endowed value, which was tax exempt. Although opportunities for higher before-tax yields on other assets were foregone, and inflation lessened some of the value of the endowed fund, the payoff from tax avoidance was sufficient to attract a number of people to the insurance market.

The deductibility of premiums from taxable income for endowment policies was made illegal by 1951, and the market responded rapidly to the legislation, as premium collections fell over 46 percent in real terms. Thus, the time series under consideration begins after the normalization of the market.[2]

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[2]Subsequent legislative measures, revised mortality tables, and other events were not viewed as having such a drastic impact.

The Dependent Variable

The value to be explained is net real insurance in force per capita. According to the model in Chapter 3, the magnitude of this variable should be negatively related to expected inflation, but positively related to human wealth.

The variable was operationally defined as in the previous chapter, except for an adjustment made to separate the savings component of investment life insurance from the protection element. The final values were arrived at according to

$$ {}_{o}Y_{t} = \frac{\dfrac{INS_t}{CPI_t} - \dfrac{RESRVS_t}{CPI_t} - \left[\dfrac{PREMS_t}{CPIJ_t} - \left(\dfrac{RESRVS_t}{CPI_t} - \dfrac{RESRVS_{t-1}}{CPI_{t-1}}\right)\right]}{POPULATION_t} \tag{50} $$

$$ = \frac{\dfrac{INS_t}{CPI_t} - \dfrac{RESRVS_{t-1}}{CPI_{t-1}} - \dfrac{PREMS_t}{CPIJ_t}}{POPULATION_t}, \tag{51} $$

where

${}_{o}Y_{t}$ designates the net real life insurance in force per capita at time t in currency units valued at time zero;

INS_t is the reported total insurance in force in Brazil at time t;

$RESRVS_t$ represent the total policy reserves on account in Brazil at time t;

$PREMS_t$ represent the total premiums collected during year t;

CPI_t is the value of the constructed consumer price index on December 31 of year t;

$CPIJ_t$ is the value of the constructed consumer price index on June 30 of year t; and

$POPULATION_t$ is the total population of Brazil, estimated for year t.

The latter version (51) more clearly shows that because the current insurance premium contains a component for cash value contribution, subtracting the entire current premium amounts to subtracting the increase in cash value as well as the cost of protection against premature death. To avoid double counting, the policy reserves were lagged one period before they were also subtracted from insurance in force, yielding the desired series of net real life insurance in force per capita. An index of these values was then formed, with the first value in both the pre and postindexing periods set equal to one. The reason for transforming Υ into an index, which will be denoted Ψ, will be explained later in the section on estimation technique.

Explanatory Variables

A number of variables have been hypothesized to influence life insurance demand. Among these are socio-demographic data and financial-economic data. Variables of the socio-demographic variety whose effect upon life insurance demand has been studied include population, number of marriages and births, mortality rates, average age of population, years of education, number of urban and rural households, ethnic

or racial origin, political climate, nationality, and geographic location. Financial-economic variables that have been related to life insurance purchases include the real and nominal rates of interest, rates of return on alternative assets, amount of nonproperty income, amount of nonhuman wealth, expected rate of inflation, supply price of insurance, level of risk aversion, seasonalities, and marketing intensity.

Several studies have been conducted to determine empirically the importance of these variables as they relate to life insurance.[3] Many of these variables contribute to the explanatory power of cross-section tests but have little worth when using aggregate data in time-series analysis (e.g., ethnic or racial origin, nationality, geographic location, years of education, etc.). Other of these variables have demonstrated insignificant explanatory power when annual data are used (e.g., seasonalities, marketing intensity, marriages, etc.).

The problem facing the analyst is to select from the remaining variables those which best facilitate the estimation process without obscuring economic relationships or spawning statistical mischief. Friedman (1957, p. 231) has observed that the inclusion of only a few explanatory variables does not necessarily present a serious problem:

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[3]References to these studies are provided in footnote 10 of Chapter 1.

> It is tempting to make a virtue of necessity by asserting that the consumer is a complex creature who is influenced by everything under the sun and hence that only an analysis in terms of a large number of variables can hope to extract a consistent pattern from his behavior. In fact, the necessity of introducing many variables is a sign of defeat and not of success; it means that the analyst has not found a truly fruitful way of interpreting and understanding his subject matter; for the essence of such a fruitful theory is that it is simple.

A problem may occur, however, if a variable which should be included is omitted. This can introduce specification bias in the estimated coefficients of the variables.[4] Thus, careful judgment must be exercised in selecting the variables to be included in and excluded from the model.

In virtually every empirical analysis that has been published, income (in nominal or real terms) in some form (permanent, transitory, disposable, gross, etc.) has been a dominant explanatory factor. In the case of Brazil, no studies were encountered which attempted to determine the importance of this factor. However, the evidence from other countries, coupled with the theoretical implications of Chapter 3, led to the inclusion of income as one of the variables to be tested for the Brazilian case. Another variable which was hypothesized to be of particular importance was inflation, especially in Brazil where it has reached high levels for extended periods of time. Theory has indicated that it may be the most important factor in determining the

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[4]See Theil (1971, pp. 549-551) for a discussion of this problem.

actual cost of insurance protection, and because inflation is the major subject of this study, its inclusion in the model was warranted.

Other variables, such as the number of live births, real rate of interest, and nominal rate of interest, which have shown some value as determinants of aggregate demand in other studies, were not included. The specification of the model was in per capita terms. Thus, data were divided by estimated population, which implicitly includes the number of live births net of deaths. Government control of the interest rate ceilings on financial assets and loans rendered data on interest rates of dubious value. For several years, the interest rate was controlled at twelve percent, even when inflation was almost nine times higher. Predictably, investors turned to commodities and real estate[5] to hedge against value losses induced by inflation. It was therefore felt that a better indicator of alternative yields on assets was the inflation rate expected to prevail. Another variable which might have been of some value in the analysis is that of non-human wealth. Because the author was unable to obtain data on this variable, it was excluded from the model. Other variables of possible importance may also have been excluded from the analysis. Accordingly, it will be impossible to rule out the presence of specification bias in the estimated coefficients of the variables.

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[5]The flight from financial assets is discussed in Chacel et al. (1970).

Inflation Expectations

The explanatory variable of primary interest in this study is that of anticipated inflation. Two approaches have been taken in the literature to estimating the values of this variable. The first, and by far less common approach, involves treating the variable as an observable.[6] It requires the existence and use of detailed survey data of public opinion regarding the rate of inflation expected to prevail during some future period. Often the survey data are merely qualitative (in the sense that they lack dimensionality) and a number of assumptions must be made in order to convert these data into an expected inflation rate series.

The second approach which is taken (and which must be taken when survey data on inflation expectations are not available) is to postulate a scheme for generating expectations in terms of observable variables and use this hypothesized scheme in place of the unobserved expected rate of inflation. When this approach is followed, the researcher ends up testing not only the hypothesis of interest but also the validity of his hypothesized scheme for generating expectations of inflation. Because these two hypotheses are tested simultaneously,

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[6] Turnovsky (1970), Pyle (1972) and Gibson (1972) have published articles based on survey data from the Philadelphia Bulletin, while De Menil and Bhalla (1975) used data from the Michigan Survey of Consumer Finances. An article by Carlson and Parkin (1975) develops expectations based on data collected in Great Britain by periodic Gallup Polls.

there is no way of knowing whether each subhypothesis taken alone would be vindicated. However, this latter approach is the one which must be taken here because there are no survey data on expectations of inflation in Brazil that span the periods of concern. Moreover, when survey data are used, they also involve the assumption that reported and actual expectations are the same. This rather tenuous assumption notwithstanding, it is somewhat comforting to know that survey data which have been used in Great Britain and the United States seem to corroborate some of the methods commonly used in generating inflation rate expectations based on observable data. Indeed, use of direct surveys has often yielded predictors of inflation that are no better and little different than simple extrapolative models based on past inflation rates (Elliott, 1977).

In this chapter it is assumed that the consumer forms his expectations of future inflation rates on the basis of past perceived rates. A common procedure is to specify an estimator based on a distributed lag model,[7] in which past values of inflation are "weighted" in some fashion to give a predicted value. A structural specification is usually imposed upon the weights of past observed inflation rates to avoid losing too many degrees of freedom and to avoid problems of multicollinearity among each of the lagged values. The most popular of

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[7]A lucid discussion of these models is given by Maddala (1977, Ch. 16).

these models is the geometric lag model wherein the weights attached to past data decline geometrically with time. This specification is often justified by summoning the logic of ''adaptive expectations,'' in which expectations are modified in accordance with the most recent forecast error.

The statistical model used to estimate expectations of inflation in this section is based on the Delayed Information Hypothesis advanced by Choate and Archer (1975). This model was selected for three reasons: (1) unlike other distributed lag models, this model does not contradict the Fisher assumption of rational avoidance by lenders and borrowers of wealth losses occasioned by changing price levels; (2) this model performs better when the historical time series of the actual inflation rate is nonstationary; and (3) the specification of this model is supported by the independent research of Carlson and Parkin (1975) based on survey data collected over a period of fourteen years in Great Britain.[8]

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[8]Carlson and Parkin concluded that when the inflation rate is high (as it has been in Brazil), expected inflation may be viewed as being generated by an error-learning process in which the previous two errors are important, suggesting that people look at both the rate of inflation and its rate of change in forming their expectations. This is precisely the basis for the Delayed Information Hypothesis. They also found that variables such as wage-price guidelines, large highly publicized wage settlements, indirect tax changes and political factors appeared to have no significant effect upon expectations of inflation.

In brief, the Delayed Information Hypothesis holds that rational consumers will accept delayed data for use within an ex ante optimal forecasting model.[9] Assuming that a given percentage price change is disseminated at the constant rate $(1-\lambda)$, $0<(1-\lambda)<1$, over time, the inflation rate perceived to have occurred in period t is j^p_t, where

$$j^p_t \;=\; \sum_{n=0}^{\infty}(1-\lambda)\lambda^n j_{t-n} \;. \tag{52}$$

Because of the general nonstationarity of the j^p series, the Box-Jenkins "near-optimal" predictor[10] is employed to relate perceived and expected inflation rates according to

$$j_{t+1} \;=\; j^p_t \;+\; j^p_t \;, \tag{53}$$

where j_{t+1} is the rate of inflation expected to prevail between time t and time t+1 and Δj^p_t is the first difference or price acceleration term for the perceived series. The Delayed Information Model results from substituting (52) into (53) to obtain

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[9]The authors state (p. 676) that ". . . delays (on price change data) are quite rationally accepted by lenders and borrowers because, while the benefits of improved data input for forecasts diminish at the margin, the marginal cost of acquiring more timely price change data for the full range of commodities increases as delays are reduced and perhaps becomes infinitely high if the search for all prices of all commodities is required to obtain perfect information at all points in time."

[10]Box and Jenkins (1962) empirically justify their "near optimal" predictor, which is a simplification of their more general forecasting model. Under nonstationarity, the "near optimal" predictor does closely approximate the performance of the general model.

$$j_{t+1} = \sum_{n=0}^{\infty}(1-\lambda)\lambda^n(j_{t-n} + \Delta j_{t-n}). \tag{54}$$

The values of j_{t-n} and Δj_{t-n} are observable, but the value of λ is generally not known a priori. Later a search procedure for estimating the appropriate value of λ will be described. The higher the value of λ, the slower the decay in the lag distribution. The mean lag of the distribution is $\lambda/(1-\lambda)$ indicating that half of the total effect of past rates of inflation upon the expected future rate of inflation will be concentrated in the first $\lambda/(1-\lambda)$ years, while the remaining effect will be distributed with declining importance over the rest of the past years.[11] While in theory this lag structure incorporates data that go back infinitely in time, when λ is as high as 0.8 (indicating a mean lag of four years), inflation rates occurring beyond the past twelve years have an almost negligible impact upon inflation expectations generated by the model. Of course, for lower values of λ, the importance of data that far back into the past is even less.

Next, the kind of data to be used in the model for approximating inflationary expectations must be decided. A consumer price index is perhaps the best source for determining the rates of price inflation that are of primary concern to the average consumer. The principal

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[11] See Griliches (1966) for a derivation of the mean lag statistic.

motive for carrying life insurance is to provide financial protection for the policy beneficiaries against the consequences of premature death of the insured.[12] The survivors will presumably expend much of their wealth for durable and consumption goods and services, whose prices are reflected in the consumer price index.

In Brazil there is no nationwide consumer price index; rather, there are separate indexes for some of the major population centers. The most prominent of these indexes, and most widely quoted, are those of Brazil's two most heavily populated cities, Rio de Janeiro and São Paulo. A geometric average of the two indexes was prepared according to

$$\mathrm{CPI}_{RJ}^{.4} \cdot \mathrm{CPI}_{SP}^{.6} = \mathrm{CPI} \qquad (55)$$

to form an overall consumer price index, where CPI denotes consumer price index and the RJ and SP subscripts refer to Rio de Janeiro and São Paulo, respectively. The weights of .4 and .6 were based on each city's approximate relative share of the life insurance market.[13] Because these two population centers account for most of the insurance sold throughout Brazil, the consumer price index based on only these two cities is deemed to be a fairly close approximation of the actual

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[12] A number of consumer surveys have substantiated this claim.

[13] The relative market shares were reported in various issues of the Statistical Annual of Brazil.

rates that the consumer might consider relevant. Moreover, there is extremely high positive correlation among all of the indexes available.

The rate of inflation (in percent) in each year was simply calculated by taking the difference between year-end values of the consumer price index for that year and the previous year and dividing it by the index of the previous year, according to[14]

$$j_t = 100 \cdot \left[(CPI_t - CPI_{t-1})/CPI_{t-1}\right]. \tag{56}$$

Insurable Human Wealth

The theoretical relationship between rational life insurance demand and human wealth derived in the preceding chapter was stated in terms of the unobservable present certainty-equivalent value of all future disposable personal wage income an individual expects to earn during his (uncertain) lifetime. This formulation follows the theoretical contributions of Yaari (1965), Hakansson (1969), Fischer (1973) and Richard (1977), and was explained at length in an article by Aponte and Denenberg (1968). However, it may be assumed that this essentially unobservable variable is related to observable data on present and past

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[14]In working with any data series, it is recognized that serious errors and biases may be included in the data. However, in the absence of anything better, the data published in various issues of Conjuntura Econômica were used. For the years prior to 1948, changes in the Rio de Janeiro consumer price index were used in isolation since no data were available for São Paulo.

values of real personal disposable income per capita.[15] In the empirical research reported in this chapter, it was postulated that the unobserved and observed data were closely (and positively) correlated to each other. To avoid transitory fluctuations in the observed data, the series was smoothed by taking a three year moving arithmetic average of the personal disposable income data published by the Conjuntura Economica. Since income is a flow (as opposed to a stock) variable, the yearly totals were deflated by mid-year levels of the constructed consumer price index and then divided by the population estimates contained in the aforementioned publication. Because the absolute level of this particular indicator for insurable human wealth is not of concern to the theoretical propositions derived in this dissertation,[16] an index of these values was

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[15]This assumption appears reasonable if people relate future income levels to present levels; i.e., if when income rises, people expect their income to maintain the higher level as a base for the trend.

[16]An alternative interpretation may be placed upon the smoothed disposable income series which treats it as a proxy for "permanent income." Friedman (1957) claims that it is this latter variable upon which consumption decisions are based. Hence, if life insurance is viewed as other consumption goods, the relationship between disposable personal income and life insurance purchases may derive from another source, namely, the size of the consumer's expected yearly "permanent" budget. This interpretation would render itself directly to the calculation of "income elasticities." The permanent income hypothesis was implicit or explicit in all the published empirical studies encountered by this author.

In the previous chapter the consumer budget was not considered to be constraining with regard to desired levels of life insurance. It was implicitly assumed that the consumer could adjust the pattern of available income (through loans, if necessary) deriving from human wealth sufficiently to be able to afford his desired level of term insurance. (This assumption would be less plausible if it had not been assumed that the

devised for both of the periods (pre and postindexing) under study where a value of unity was assigned to the first year of both series, and succeeding variations in the index corresponded in exact proportion to relative changes in the deflated disposable personal income per capita series.

One rather serious problem was encountered with the data on disposable personal income. The system used for calculating the statistical series was modified after 1973. The revised (and presumably the preferred) formulation was applied only to the period from 1965 to the present. An analysis of the two series over an extended length of time revealed that a simple splicing procedure would be inappropriate. However, in publishing the revised data series, two "benchmark" years were also included, namely 1949 and 1959. The procedure used to estimate the income series for the missing years, compatible with the data generated by the revised income accounting procedures, was as

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rational consumer separates the protection from the savings component of life insurance, since investment life insurance policies can place a sizable dent in almost anybody's budget.) Fama and Schwert (1977) have noted that if selling oneself into slavery is illegal, there may be a limit of perhaps the equivalent of two year's income beyond which the insured may be unable to borrow. These constraints have been studied elsewhere, and while they are of theoretical interest, they are probably of little importance in constraining the levels of rational insurance purchasing to the typical insured. Although the author prefers the interpretation and significance placed upon the real disposable personal income per capita index deriving from the theoretical formulation of Chapter 3, the reader is free to interpret the empirical relationships estimated for this variable in accordance with the Friedman explanation.

follows:

$$D_t = \frac{(t-1949)}{10} \cdot D_{1959} + \frac{(1959-t)}{10} \cdot D_{1949}, \qquad (57)$$

where D_t represents the old data shift multiplier for year t, and $D_{1949} = DPI^{new}_{1949}/DPI^{old}_{1949}$, $D_{1959} = DPI^{new}_{1959}/DPI^{old}_{1959}$ and DPI designates the reported disposable personal income. The resulting shift multipliers were then applied to the old data series for the missing years to make the two series of conformable magnitudes. This procedure was repeated for the interval between 1959 and 1965. Adjustments were then made to present the entire (revised) series in real values per capita. The data were then smoothed, as explained above, and converted into an index.

Estimation Procedure

To measure the influence of the explanatory variables on the dependent variable, a multivariate linear regression model was specified of the following form:

$$\Psi_t = \beta_1 \chi_{1t} + \beta_2 \chi_{2t} + \varepsilon_t, \qquad (58)$$

where Ψ_t is the dependent variable (in the form of an index) measured at time t, χ_{1t} is the expectation at time t for future inflation rates, in percent, and χ_{2t} is an index indicating relative levels of insurable human wealth. It was reasoned that since the per capita disposable personal income figure for any year was always below the minimum

of the range where life insurance purchases generally begin to take place, (a reflection of the extreme inequality of income distribution), then the absolute levels of the data were moot. Rather, the importance of the data stems from their (presumed) ability to indicate changes and trends in the unobservable quantity of concern,[17] insurable human wealth.

The regression equation (58) is in simplified form. The reader will recall that Ψ, χ_1 and χ_2 are themselves the results of the mathematical transformations identified earlier. The estimation procedure for Ψ and χ_2 involves a straightforward application of the formulas to the observable data (e.g., population, consumer price indices, nominal disposable personal income, insurance in force, premiums and reserves). The estimation procedure for χ_1, however, involves the application of weights to realized inflation rates going back infinitely into the past. While some of these rates were recorded, and hence may be considered observed, before a certain point in time the rates were not tabulated and are therefore not available to be included in the estimation procedure. Even if they were available, the market basket of commodities upon which they are based would have undergone tremendous

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[17]If Friedman's "permanent income" hypothesis is substituted for that of insurable human wealth, then the absolute level of real disposable personal income per capita is much more likely to approximate the level of permanent income. However, it still would not be in the relevant insurance purchasing range, when taken in the aggregate. Although one might expect a linear relationship to hold, even in the irrelevant range, theory has not yet dealt with this issue.

revisions, and a historical sequence of these rates would be of dubious validity.

Several approaches have been taken in the literature to deal with this problem. The approach taken here is a modified version of the direct estimation technique first suggested by Klein (1958).[18] Expanding the χ_1 term of equation (58) by substituting the information contained in (54) gives

$$_o\Psi_t = \beta_1 \sum_{n=1}^{\infty}(1-\lambda)\lambda^n(j_{t-n} + \Delta j_{t-n}) + \beta_2\chi_2 + \varepsilon_t \tag{59}$$

This can be expanded further by separating the variable representing inflation expectations into two parts as follows:

$$_o\Psi_t = \beta_1(1-\lambda)\sum_{n=0}^{t-1}\lambda^n(j_{t-n}+\Delta j_{t-n}) + \beta_1(1-\lambda)\sum_{n=t}^{\infty}\lambda^n(j_{t-n}+\Delta j_{t-n}) + \beta_2\chi_{2t} + \varepsilon_t \tag{60}$$

Here the first term may be computed from actual observations, given a value of λ. However, the second term involves rates of inflation that occurred before period one and which (Klein assumed) were not observable. His solution is to rewrite $n-t = \tau$ and rewrite the second term of χ_1 as

$$\lambda^t\beta_1(1-\lambda)\sum_{\tau=0}^{\infty}\lambda^\tau(j_{-\tau} + \Delta j_{-\tau}) = \lambda^t\eta_0 \tag{61}$$

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[18] The presentation here closely follows that given in Maddala (1977).

where $\eta_0 = E[\Psi_0] = \beta_1(1-\lambda)\sum_{n=0}^{\infty}\lambda^n(j_{-n} + \Delta j_{-n})$. (62)

Thus (58) can be rewritten as

$$ {}_o\Psi_t = \beta_1 Z_{1t} + \eta_0 Z_{2t} + \beta_2 X_2 + \varepsilon_t, \tag{63} $$

where

$$ Z_{1t} = (1-\lambda)\sum_{n=0}^{t-1}\lambda^n(i_{t-n} + \Delta i_{t-n}), \tag{64} $$

$$ Z_{2t} = \lambda^t, $$

and η_0 is a parameter which accounts for the truncation remainder. The coefficients of (63) are estimated by first constructing the variable Z_{1t} and Z_{2t} for various values of λ, $0<\lambda<1$, and then performing an ordinary least squares procedure. The value of λ for which the residual sum of squares is at a minimum is the maximum likelihood value of λ, and the resulting estimates for β_1, η_0, and β_2 are also maximum likelihood estimates, where the coefficients of concern, β_1 and β_2, are consistent estimators.

Implicit in the Klein procedure is the assumption that the data series for the variables of concern begin at the same time. In the case of this study, the inflation rate series for the first period goes back to 1945, whereas the life insurance in force series begins in 1951. The Klein procedure involves discarding data for a six year period in the first sample. In the second postindexing sample, following the Klein

procedure would lead to discarding the inflation data collected from 1945 through 1967, a twenty-three year period.

To remedy this inefficient use of available data, the Klein method was adjusted so that these earlier observations would be included in the X_{1t} term of (63), while assigning the unobserved data to the truncation remainder. For the preindexing sample, the transformation was expressed as

$$ {}_{o}\Psi_{t} = \beta_{1}(1-\lambda)\sum_{n=0}^{t+5}\lambda^{n}(j_{t-n} + \Delta j_{t-n}) + \eta_{0}\lambda^{t+6} + \beta_{2}X_{2t} + \varepsilon_{t}, \quad (66) $$

and the transformation for the second postindexing sample was expressed as

$$ {}_{o}\Psi_{t} = \beta_{1}(1-\lambda)\sum_{n=o}^{t+22}\lambda^{n}(j_{t-n} + \Delta j_{t-n}) + \eta_{0}\lambda^{t+23} + \beta_{2}X_{2t} + \varepsilon_{t}. \quad (67) $$

Next, the propositions derived in Chapter 3 were recalled, and reformulated in terms suitable for testing the econometric model specified above. The propositions lead to expectations concerning the signs of the coefficients to be estimated. In the case of the variable representing inflationary expectations, it is recollected that the rate of inflation anticipated should be negatively related to purchases of net real life insurance protection. Thus, the coefficients β_1 and (therefore) η_0 can be expected to carry negative signs. Of course, the amount of insurance in force is expected to be positively related to insurable human wealth, and so β_2 is hypothesized to be positive.

According to the theoretical models developed in the second and third chapters, these relationships should hold in both the preindexing and postindexing periods. These propositions can be summarized in operational form as:

Preindexing Period (1951-1967):

H_{01}: $\beta_1 \geqq 0$ $\qquad$ H_{02}: $\beta_2 \leqq 0$

H_{A1}: $\beta_1 < 0$ $\qquad$ H_{A2}: $\beta_2 > 0$

Postindexing Period (1968-1976):

H'_{01}: $\beta_1 \geqq 0$ $\qquad$ H'_{02}: $\beta_2 \leqq 0$

H'_{A1}: $\beta_1 < 0$ $\qquad$ H'_{A2}: $\beta_2 > 0$

where H_o indicates null hypothesis and H_A indicates preferred hypothesis. The usual procedure is to tentatively accept the null hypotheses unless evidence is presented which makes their validity extremely suspect.

In a first test of the model, the possibility of serial correlation among the successive residuals arose. The Durbin-Watson test statistic for autocorrelation was located in the indeterminate range at the 1 and 5 percent levels of significance for the preindexing period. Because of the low number of observations in the postindexing period, it was also uncertain whether autocorrelation existed. Therefore, to safeguard against the possibility that the least squares estimators are not best linear unbiased estimators (BLUE), are asymptotically

inefficient, and exhibit biased estimates of the standard errors of the coefficients, the Cochrane-Orcutt procedure was performed to adjust for any first order autoregressive process that might be present.[19] This amounted to transforming the variables Ψ_t, Z_{1t}, Z_{2t} and X_{2t} into a quasi-first-difference equations as follows:

$$\Psi_t^* = \Psi_t - \rho\Psi_{t-1}$$

$$Z_{1t}^* = Z_{1t} - \rho Z_{1t-1}$$

$$Z_{2t}^* = Z_{2t} - \rho Z_{2t-1}$$

$$X_{2t}^* = X_{2t} - \rho X_{2t-1}$$

A regression was then performed on the transformed variables

$$\Psi_t^* = \beta_1 Z_{1t}^* + \eta_o Z_{2t}^* + \beta_2 X_{2t}^* + e_t . \qquad (68)$$

A search procedure was performed over different values of λ, while the Cochrane-Orcutt transformation and estimation procedure were applied to each of the equations resulting from the search procedure. The equations which minimized the sum of squared errors were selected for the final estimation.

The resulting estimators from the application of the model to the data (where the t statistics are placed in parentheses) were:

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[19] If serial correlation does, in fact, exist, it is reasonable to suspect an autoregressive process of first order since monthly and quarterly data are not used. Thus, $\varepsilon_t = \varepsilon_{t-1} + e_t$, where e_t is independently and normally distributed.

Preindexing Period (1951 - 1967)

$$\hat{\Psi}^*_t = -.014632Z^*_{1t} - 201.994Z^*_{2t} + 1.5235\chi^*_{2t} \qquad \rho = .55 \quad \lambda = .6$$
$$\quad (-4.1166) \qquad (-1.7692) \qquad (10.56)$$

$R^2 = .6893$ F = 14.423 Durbin-Watson = 1.7574

Postindexing Period (1968 - 1976)

$$\hat{\Psi}^*_t = -.019511Z^*_{1t} - 30739.90Z^*_{2y} + 1.6450\chi^*_{2t} \qquad \rho = -.27 \quad \lambda = .6$$
$$\quad (-8.6929) \qquad (-3.5967) \qquad (44.12)$$

$R^2 = .9969$ F = 797.62 Durbin- Watson = 2.7165

The first item of interest is the signs associated with the parameters of concern, β_1 and β_2 The signs of each of these coefficients for both the preindexing and postindexing periods are those predicted by the relevant theory, and each of the coefficients is significantly different from those values stated in the respective null hypothese at the 1 percent level of significance (and less). Therefore, all four of the null hypotheses are summarily rejected in favor of their counterpart preferred hypotheses.

Any problem of serial correlation among the residuals, if indeed there were any problems, appear to have been eliminated, as the Durbin-Watson test statistics are all in the desired ranges. The F statistics also indicate that the model as a whole has explanatory power for the dependent variable. For both the preindexing and postindexing periods, the maximum likelihood estimator of λ was .6, indicating that the mean

lag associated with using past inflation rates and their changes for generating inflation expectations was about 1.5 years. The value of ρ for the period prior to indexing was about .55 whereas its value for the period of indexing was -.27. The signs of these parameters appear in order since the earlier period was marked by inflation rate series with long trends, whereas in the latter period, inflation rates tended to oscillate.

The elasticities of net real life insurance in force per capita with respect to expected inflation and insurable human wealth (or alternatively, permanent income per capita), before and after indexing, can be calculated by multiplying the estimated coefficients by the averages of the explanatory variables of concern (expected inflation rate and insurable human wealth index) and dividing these products by the average levels of the index representing net real life insurance in force per capita. This operation resulted in the following elasticities:

	Inflation Elasticities	Income Elasticities
Preindexing	-.43	1.55
Postindexing	-.29	1.31

The interpretation of these elasticities is straightforward. In the years prior to indexing, a relative increase of 1 percent in the expected rate of inflation (where the average expectation for the period was calculated at 35 percent; thus, a 1 percent relative rise translates

into an increase in expected inflation of 0.35 percent) can be expected to result in 0.43 percent less net real life insurance in force per capita. Therefore, a 1 percentage point rise in inflation is expected to be accompanied by a 1.23 percent decline in net real life insurance in force per capita. On the other hand, a 1 percent increase in normalized real disposable income per capita was accompanied by an increase in insurance of 1.55 percent.

In the postindexing period, a 1 percent relative rise in the expected inflation rate led to a decline in net real life insurance in force per capita of about .29 percent whereas when a 1 percentage point rise in inflation was expected, insurance purchases fell by 1.16 percent. A 1 percent increase in normalized real disposable personal income per capita was accompanied by an increase in net real life insurance in force per capita of 1.31 percent.

There is little surprise in these calculated elasticities. Although the "income" elasticities are greater than unity, these results are not difficult to explain. Certainly part of the magnitude of the elasticities can be accounted for by the fact that as insurable human wealth rises, insurance coverage is also likely to rise to protect against the possibility of a larger loss of income if the breadwinner dies prematurely. Another portion of the magnitudes of the elasticities can be explained by noting that as a developing country, Brazil had a growing middle

class. Urbanization was taking place and many multinational firms began or expanded their operations during these years. These factors combined to produce a growing "relevant" insurance market.

The expected inflation elasticities also are of reasonable magnitudes, and show only small differences between the two periods. The inflation elasticity of the first period is slightly larger than that of the postindexing period. Perhaps the greater transactions and nuisance costs[20] precipitated by inflation during the preindexing period were responsible for the relatively greater elasticity. Another reason proffered is that although indexing has resulted in virtually no reduction in the net expected (real) cost per unit of expected (real) benefits for term insurance, it has resulted in substantial declines in the net expected (real) cost per unit of investment insurance benefits. To the extent that insurance consumers did not separate the essentially different components of life insurance policies, obtaining life insurance protection could have appeared relatively less costly than before. The difference is small, however, especially when changes in inflation rate expectations are viewed in absolute terms.

It is perhaps more illuminating to attempt to explain why these elasticities are so similar. One of the reasons that the elasticities (especially when changes in inflation expectations are viewed in

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[20]For an explanation, see concluding section of Chapter 2.

absolute terms) are so close might be that by the time index-linked investment policies arrived (which offer a secure 3.5 percent real rate of return on the policy reserves) other investment vehicles of similar risk were offering more than a 6 percent guaranteed rate of return. The enhanced absolute attractiveness of indexed life insurance policies was overshadowed by the possibilities of saving elsewhere at double the rate of interest.

Another reason might be that index-linked policies account for a small portion of the total policies sold in Brazil. In interviews with marketing executives and actuaries of the large Brazilian insurers, it was learned that only 10 to 30 percent of the policies sold were index linked. Indeed, one of the largest life insurance companies did not even offer indexed policies!

A different interpretation of the "inflation expectation" elasticities is that they are merely a technical result. Noting that a 1 percentage point rise in "expected inflation" was accompanied by a decrease in net real life insurance in force per capita of slightly more than 1 percent in both periods, and noting that the estimator for the expected rate of inflation is simply based on present and past rates of realized inflation, it may be argued that if insurance purchases are slow to adjust to higher price levels, the effect could be similar to actively and consciously adjusting insurance in force according to expected levels of

inflation.[21] Other model specifications tried were unable to totally eliminate ambivalence about the source from which the effect arose. However, an absolute change of 1 percent was associated with greater than 1 percent declines in net real life insurance in force per capita. Hence perhaps both explanations may in fact have some validity. The important result of the empirical analysis, however, is that inflation, whether realized or expected, was incontestably related to lower net real life insurance in force per capita.

Comparison of Research Model and Results with Previous Studies

Of the five articles on inflation and life insurance sales cited in Chapter 1, two (Houston, 1960, and Fortune, 1972) did not measure the absolute amounts of insurance in force; rather, they dealt with

[21] For example, if prices rise by 1 percent while insurance in force remains constant, real insurance in force will fall by approximately 1 percent. Then suppose that prices subsequently stabilize at the higher price level, or even return to their former level, while insurance in force gradually readjusts upward by 1 percent to "catch up" for the losses incurred by the previous period. If inflation were anticipated correctly at 1 percent for the first period and 0 percent (or less) for the second period, a 1 percent expected rate of inflation would be linked to a 1 percent decline in real insurance in force, while a 0 percent or negative expected rate of inflation will be accompanied by a 1 percent or greater increase in real insurance in force. Thus, the hypothesized relationship of rational insurance purchasing to expected inflation could be replicated by a delayed adjustment of insurance in force to the price level. If inflation rates show accelerating trends or decelerating trends, or if they oscillate widely, those patterns could give rise to the observed phenomemon. This adjustment lag could be a mere technical result of tying group insurance to income, which may not be adjusted to inflation except on an ex post basis, or it could be due to delayed reporting of inflation rates.

preferred amounts of investment life insurance or reserves relative to total insurance. Thus, they do not readily compare to the model estimated here.

The remaining three articles contain statistical evidence which is of relevance to that presented in this study. Of course, the most apparent difference is that Hofflander and Duvall (1967), Neumann (1967) and Fortune (1973) all contain tests based only on the American data, whereas the tests conducted in this study used Brazilian data. Moreover, in none of the previous papers were tests performed over both preindexing and postindexing sample data, since indexation of American life insurance policies has not yet occurred on a widespread basis. Fortune's data covered a later period than that of Hofflander and Duvall and Neumann.

All four studies contained tests where life insurance in force for term, investment, or both kinds of policies were specified as the dependent variables. The Hofflander and Duvall, and the Neumann models used annual nominal aggregate life insurance in force as the dependent variable, while Fortune used quarterly real per capita life insurance in force, net of policy reserves. Fortune's dependent variable compares most closely with that specified here, where an index of annual real per capita life insurance in force, net of policy reserves and premiums, was used.

Several models have been used to estimate the inflationary expectations that may influence life insurance purchases. Hofflander and Duvall used the absolute level of the consumer price index in their model,[22] while Neumann used the consumer price index in both absolute levels and first differences. In both studies, inflationary and price expectations were generated by a distributed lag model in accordance with the adaptive expectations framework, with geometrically declining weights assigned to data from past years. A Koyck transformation was applied to the equations where the dependent variable lagged one year became one of the explanatory variables. Fortune estimated the effect of anticipated inflation by using the Index of Consumer Sentiment, constructed by the Michigan Survey Research Center. He found this index negatively correlated to the expected quarterly rates of inflation reported by Feldstein and Eckstein (1970), and used it as a proxy for expected inflation rates. The Brazilian study employed a near-optimal predictor in the context of the Delayed Information Hypothesis. A modified version of Klein's direct estimation procedure was used to generate the expectations.

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[22]It is not altogether clear from their article whether Hofflander and Duvall used the absolute level of the consumer price index or changes in the index. However, during telephone conversations with the authors it was disclosed that the absolute level of the index was used.

Each of the models included other explanatory variables. One variable included in all the models was income. Hofflander and Duvall used nominal personal income; Neumann used nominal disposable personal income; Fortune used the logarithms of real per capita quarterly income data of households, and the Brazilian test used an index of three year moving averages of real per capita disposable personal income.

Of the three tests performed on the American data, Fortune's model appears to conform most closely to the theoretical relationships derived in Chapter 3. Hofflander and Duvall found a negative relationship between expected inflation and life insurance sales in nominal terms, whereas Neumann found no significant relationship between these two variables. Fortune found the rate of expected inflation imputed to the Index of Consumer Sentiment to be significantly and negatively related to real net per capita life insurance in force, as did the Brazilian study. It is felt that much of the divergence in the results and their implications can be traced to the differences in the models, and, perhaps to a lesser extent, to the different data bases. Because of inflation's high correlation with nominally-valued insurance in force and income, tests performed using variables denominated in nominal terms may not capture the underlying economic relationship of concern. Moreover,

use of nominal variables does not incorporate the widely used assumption that allocation decisions are based on real quantities in the absence of future price speculation. Thus, it would appear that the Fortune and Brazilian studies are comparable, and each serves to confirm the hypothesized relationships of this study.

Summary and Conclusions

The problem dealt with in this dissertation was expressed by a single question: should index-linked life insurance contracts be offered? Whether or not economic conditions and institutions are compatible with the introduction of indexed life insurance, the answer to the question depends upon: (1) the extent to which inflation affects the cost of life insurance protection; (2) the degree of consumer demand sensitivity to inflation-produced changes in life insurance cost; and (3) whether index-linked contracts can effectively mitigate any adverse effects of inflation upon life insurance costs and sales. The Brazilian experience with inflation and indexation was summoned to provide empirical content to the investigation.

In measuring the direction and extent of inflation's impact upon the cost of life insurance protection, it was shown that unless care is taken, the (mis)application of life insurance costing methods currently in vogue can lead to incorrest conclusions. A procedure appropriate for measuring the effect of inflation upon life insurance costs was

developed and applied to nonindexed policies. It was demonstrated both in theory and in practice that anticipated inflation causes the rationally perceived cost of protection through conventional policies to rise. Although this conclusion holds for both term and investment life insurance policies, the magnitude of inflation's adverse effect on the cost of insurance was considerably greater for investment policies.

Next, the issue of consumer sensitivity to expected inflation was studied. A somewhat novel theoretical model was devised for examining inflationary expectations and rational life insurance purchasing. The model permitted some advantages of a multiperiod formulation, while stated in a simple two period format. The time-state preference framework used enabled the analysis to proceed with state dependent preference functions. The treatment permitted precision with respect to the timing and incidence of expected future rates of inflation, and unambiguously demonstrated its negative direct and indirect effects upon the purchase of life insurance.

In addition to showing theoretically that a risk-averse rational consumer will decrease purchases of (real) insurance protection when inflation is expected, statistical tests were conducted on the Brazilian data base to discover if, in fact, consumers behaved as predicted by theory. Consumers were found to be sensitive to inflation by reducing their purchases of life insurance. Whether this phenomenon resulted

from deliberate or passive actions, consumers can be said to have behaved as if they were risk averse and rational.

Finally, indexation of life insurance policies was viewed both in theory and in practice. In theory, life insurance contracts could be continuously indexed so that the real cost of protection would be invariant with respect to inflation. If such were the case in practice, one could hypothesize that rational consumer demand for life insurance would be inflation insensitive, other things the same. However, in Brazil the indexation of life insurance permits only annual adjustments of policy values. Therefore, the rationally perceived cost of life insurance protection increases with anticipated inflation, and (real-valued) sales can be expected to decline. These latter two hypotheses were statistically confirmed by the Brazilian experience. Thus, while it is possible that indexation can eliminate the adverse effects of inflation upon life insurance, this aim has not been achieved in Brazil.

Divergent interpretations and explanations could be offered regarding the similar relationships observed between inflation and life insurance sales before and after indexing. At stake is whether a high degree of rationality is imputed to the Brazilian insured, who adjusts his insurance purchases to maximize his expected utility in light of expected inflation, or if the Brazilian insured is merely acted upon or reacts automatically, where levels of insurance in force are

adjusted as a technical result of some institutional arrangement or phenomenon.

At this point two items may be cited which lend support to the attribution of rationality to the Brazilian consumer of insurance. First, it is recalled that when a loophole was spotted in the tax provisions of 1950, consumers reacted strongly by substantially increasing their insurance purchases. Second, unlike in the United States and elsewhere, insurance in Brazil is not marketed intensively. It is not a "sold good," at least not to the degree that it is in North America. Rather, it more closely approximates a "sought good," and is made available to those who seek it. This marketing approach has resulted in comparatively shallow penetration into the potential insurance market of Brazil. In fact, recently when a Japanese owned and oriented company set up operations in Sao Paulo, seeking primarily a clientele among Brazilians of Japanese origin, it vaulted from nothing to twelfth largest underwriter in the country in a matter of about two years through intensive, and heretofore unpopular, door to door marketing. Because in Brazil insurance is more likely to be a sought good, the patrons of insurance are likely to come from a more sophisticated class of consumers, who perhaps are more inclined to act rationally.

One conclusion appears incontestable. Inflation (expected and/or realized) has been negatively related to net real per capita life insurance in force in Brazil, both before and after indexation. Unfortunately,

the Brazilian version of index-linked life insurance contracts does not allow a definitive conclusion to be reached with regard to the advisability of offering indexed policies. It can be said that the availability of indexed life insurance leaves the consumer better off, ipso facto, if only by providing an additional alternative from which to choose. However, the degree of financial uncertainty remains almost as high as without indexed policies, and the real cost of (term) life insurance protection is only marginally affected by indexation. The lack of an enthusiastic consumer response to indexed policies is therefore understandable.

However, these findings should not be misconstrued as an indictment against indexation; rather, they serve to indicate that the system of life insurance indexing used in Brazil has not been entirely successful in offsetting the adverse effects of inflation on life insurance values and sales. A continuously indexed policy, if offered, could result in rationally perceived insurance costs that are invariant to the rate of inflation. Whether such a policy could also restore consumer demand to the hypothetical equilibrium levels associated with stable prices remains purely a matter of speculation.

The inquiry of this dissertation suggests a number of areas where further research may be warranted. The theoretical models for measuring life insurance costs under inflation could be extended to include

other provisions, such as dividends, and other patterns of desired financial protection (increasing or decreasing). The models could then be applied to policies offered outside of Brazil to determine the dimensions of inflation's influence on life insurance costs elsewhere.

Another opportunity for advancing the contributions of this paper lies in expanding and generalizing the theoretical models used in examining rational life insurance purchases. Uncertainty could be extended to include not only interperiod but intraperiod timing of death, as well as the levels of future income, inflation, and investment opportunities.

The present study has emphasized the consumers' point of view. Implicit in this approach was the assumption that the consumer is primarily interested in the cost of providing financial protection. Future research could take the insurance companies' point of view. Presumably, the effects of inflation and indexation upon the long run profitability of the enterprise would be of major concern. The theoretical and statistical models could be reconstructed in terms of net (real-valued) premium revenues generated. Broadening the study to encompass the interests of society at large, the effects of inflation and indexation upon the contribution of the insurance industry to capital formation would be of importance. In Brazil, insurers often invest all of the net proceeds from the sale of indexed policies in indexed

government bonds, even though this is far in excess of the required portion. Such a transfer of capital from the long-term debt of the private sector to the public sector may radically alter the nature and degree of capital formation deriving from insurance.

APPENDIX A

FORMAL RELATIONSHIP BETWEEN INFLATION AND LIFE INSURANCE POLICY COST

To determine the effect of a change in the discount rate upon the expected net present cost of a life insurance policy, the differential $dE[NPC_k]$ is taken with respect to the discount factor, I:

$$dE[NPC_k] = \frac{dE[NPC_k]}{dI}\, dI \ .$$

Recalling the level premium term policy of Chapter 2 and differentiating its respective formula[1]

$$dE[NPC_k] = [P\sum^{k} s_{n-1}(1-n)I^{-n} - Cr\$1000\sum^{k} d_n(\tfrac{1}{2}-n)I^{-n-\frac{1}{2}}]dI \qquad (1)$$

Here it is seen that for n = 1, 2, . . . , k, the first term will be a nonpositive expression while the second will be positive. The sum of these expressions within the brackets, whether positive or negative, specifies the direction of impact of a change in the discount rate (and hence the direction of impact of a change in the expected rate of inflation).[2] To examine the effect of a change in anticipated inflation upon the cost of a term policy, the year of interest is merely indicated and the appropriate parameters substituted into equation (1). As will

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[1]The proof outlined here follows closely that of Babbel (1977). The derivation above in (1) was performed on equation (3) on page 30. The final term of (3) was dropped since it is irrelevant to a term policy, and the simplified notation of page 57 is used, where (1+i)=I.

[2]Note that the assumption is retained that if inflation is anticipated to rise, the discount factor will also rise.

be shown, the direction of change is highly sensitive to the year for which the valuation is performed.

Considered first is the effect of a change in anticipated inflation on the expected net present cost of a term policy in year one (k=1). In this case, the first expression is zero while the second expression is clearly positive; hence, a rise in the anticipated rate of inflation will unambiguously result in a higher expected cost of a term insurance policy for a one year horizon.

Considered next is the effect of a changing discount rate (or expected rate of inflation) upon the cost of a term insurance contract which is held for two years (k=2). In this case (1) becomes

$$dE[NPC_2] = [-Ps_1 I^{-2} + Cr\$1000\Sigma d_n (n-\tfrac{1}{2}) I^{-n-\frac{1}{2}}]dI \ . \tag{2}$$

After collecting terms and adjusting for a common denominator,

$$dE[NPC_2] = \left[\frac{-2Ps_1 I^{\frac{1}{2}} + Cr\$1000d_1 I + 3Cr\$1000d_2}{2I^{5/2}}\right] dI. \tag{3}$$

Since the denominator will be positive for any empirically relevant discount rate, the sign of the sum in the numerator will determine the direction of a change in $E[NPC_2]$ produced by a change in the discount rate. If the sum is positive (negative) an increase in the discount rate will produce a rise (decline) in the E[NPC] evaluated for a two year holding period.

To determine the sign of the numerator in (3) it is first assumed that the expected present value of premiums charged exactly covers the expected present value of the death protection, i.e., the E[NPC] of the policy is zero. Then, from (3) of Chapter 2 it is known that

$$\Sigma P s_{n-1} I^{1-n} = \Sigma d_n Cr\$1000 I^{\frac{1}{2}-n} \tag{4}$$

which reduces to

$$P \Sigma s_{n-1} = Cr\$1000 \Sigma d_n I^{-\frac{1}{2}}. \tag{5}$$

For the period under analysis k=2 so

$$P + Ps_1 = I^{-\frac{1}{2}} Cr\$1000 d_1 + I^{-\frac{1}{2}} Cr\$1000 d_2 ; \tag{5}$$

$$\therefore P = [I^{-\frac{1}{2}} Cr\$1000(d_1 + d_2)]/(1+s_1). \tag{6}$$

Substituting this result into the numerator of (3), the sign of the numerator is determined as follows:

$$\frac{-2Cr\$1000[d_1 + d_2]s_1}{1 + s_1} + Cr\$1000 I d_1 + 3Cr\$1000 d_2 \overset{?}{\gtrless} 0; \tag{7}$$

$$\frac{d_1 + d_1 i + 3d_2}{d_1 + d_2} - \frac{2s_1}{1+s_1} \overset{?}{\gtrless} 0. \tag{8}$$

Since the first quotient in (8) is greater than one while the second is less than one, the sum is greater than one; therefore, it is claimed that a rise in the discount rate will raise the E[NPC] of such a term policy held for two years.

In reality, however, the assumption formulated in (6) is totally implausible. Insurance premiums must exceed the death benefits in order to cover salesman's commissions, loading and investment costs, and provide a return to the stockholders. When the equality given in (6) is relaxed, the net effect of a change in the discount rate upon the cost of the two year policy can no longer unambiguously be determined, a priori, without further information regarding the size of the premium outlays.

It is known from (3) that if

$$P > Cr\$1000 d_1 I^{\frac{1}{2}}/2s_1 + 3Cr\$1000 d_2/2s_1 I^{\frac{1}{2}}, \tag{9}$$

then a term policy held for two years will fall in E[NPC] with a rise in anticipated inflation. For a typical policy the premiums charged considerably exceed the right hand side of (9); therefore, anticipated inflation will be associated with a decreasing E[NPC] of a policy that is intended to be surrendered after two years.

This analysis could be continued to determine the effects of anticipated inflation rate changes upon the cost of term policies held for various lengths of time. In general, it may be stated that the expected net present cost of a policy held beyond one year decreases with anticipated inflation. This is true whether term or investment insurance is purchased.

APPENDIX B
INFLATION AND THE COST OF INSURANCE PROTECTION

To determine the effect of a rising discount rate, $I=(1+i)$, upon the cost of life insurance protection through a term policy involves totally differentiating the cost-benefit equation of Chapter 2 (12) with respect to the discount rate.

$$\frac{E[NPC_k]}{E[PV(B)_k]} = \frac{E[PV(C)_k]}{E[PV(B)_k)} - 1 \tag{1}$$

$$d\frac{E[NPC_k]}{E[PV(B)_k]} = \frac{dE[NPC_k]/E[PV(B)_k]}{dI}\, dI \tag{2}$$

Where

$$\frac{E[NPC_k]}{E[PV(B)_k]} = \frac{\sum_{n=1}^{k} \frac{P(1/(1-DR_{a-1}))\prod_{t=0}^{n-1}(1-DR_{a+t-1})}{I^{n-1}}}{\sum_{m=1}^{k} \frac{DR_{a+m-1}(Cr\$1000)}{I^{m-\frac{1}{2}}}} \tag{3}$$

Substituting (3) into (2) and operating

$$d\frac{E[NPC_k]}{E[PV(B)_k]} = \frac{[P\frac{1}{1-DR_{a-1}}\sum_{n=1}^{k}(1-n)I^{-n}\prod_{t=0}^{n-1}(1-DR_{a+t-1})][Cr\$1000\sum_{m=1}^{k}I^{-m+\frac{1}{2}}DR_{a+m-1}]}{[Cr\$1000\sum_{m=1}^{k}I^{-m+\frac{1}{2}}DR_{a+m-1}]^2}\, dI$$

$$- \frac{Cr\$1000\sum_{m=1}^{k}(\tfrac{1}{2}-m)I^{-m-\frac{1}{2}}DR_{a+m-1}][P\frac{1}{1-DR_{a-1}}\sum_{n=1}^{k}I^{-n+1}\prod_{t=0}^{n-1}(1-DR_{a+t-1})]}{[Cr\$1000\sum_{m=1}^{k}I^{-m+\frac{1}{2}}DR_{a+m-1}]^2}\, dI \tag{4}$$

If $d[\,\cdot\,]/dI > 0$, then as I increases the $E[NPC_k]/E[PV(B)_k]$ increases, and conversely. Therefore, the problem is to determine if the derivative of the cost-benefit ratio with respect to the discount factor is

greater than zero. Since the symbols of the denominator designate real numbers, the denominator, which is squared, is positive. Hence, the concern rests with the sign of the numerator which, if greater than zero, indicates that the cost of life insurance protection increases as the discount factor rises.[1]

By factoring the level premiums out of the body of the numerator, the expressions of concern may be simplified to

$$[P\frac{1}{1-DR_{a-1}}Cr\$1000][(\sum_{n=1}^{k}(1-n)I^{-n}\prod_{t=0}^{n-1}(1-DR_{a+t-1}))(\sum_{m=1}^{k}I^{-m+\frac{1}{2}}DR_{a+m-1})$$

$$-(\sum_{n=1}^{k}I^{-n+1}\prod_{t=0}^{n-1}(1-DR_{a+t-1}))(\sum_{m-1}^{k}(\tfrac{1}{2}-m)I^{-m-\frac{1}{2}}DR_{a+m-1})] \overset{?}{>} 0 \quad (5)$$

Because the term in the first set of brackets is positive by definition, the sign of the product (and thus the sign of the derivative) will depend upon the mathematical nature of the terms in the second set of brackets. These terms may be combined as

$$\sum_{n=1}^{k}\sum_{m=1}^{k}[I^{-n-m+\frac{1}{2}}DR_{a+m-1}\prod_{t=0}^{n-1}(1-DR_{a+t-1})-(\tfrac{1}{2}-m)I^{-n-m+\frac{1}{2}}DR_{a+m-1}\prod_{t=0}^{n-1}(1-DR_{a+t-1})] \overset{?}{>} 0 \quad (6)$$

and further simplified to

$$\sum_{n=1}^{k}\sum_{m=1}^{k}[I^{-n-m+\frac{1}{2}}DR_{a+m-1}\prod_{t=0}^{n-1}(1-DR_{a+t-1})\{(1-n)-(\tfrac{1}{2}-m)\}] \overset{?}{>} 0 \quad (7)$$

$$\sum_{n=1}^{k}\sum_{m=1}^{k}[I^{-n-m+\frac{1}{2}}DR_{a+m-1}\prod_{t=0}^{n-1}(1-DR_{a+t-1})(\tfrac{1}{2}-n+m)] \overset{?}{>} 0 \quad (8)$$

To determine if (8) exceeds zero, it is necessary to substitute different values of k into the equation and solve. First, for k=1,

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[1]Note that the pivotal assumption here is that the policy premium levels remain invariant. It was pointed out earlier (in Chapter 2) that this assumption was a good approximation of the Brazilian reality.

$$I^{-1\frac{1}{2}} DR_a (1-DR_{a-1})[\tfrac{1}{2}-1+1] > 0 \tag{9}$$

Next, for k=2,

$$I^{-1\frac{1}{2}} DR_a (1-DR_{a-1})(\tfrac{1}{2}) + I^{-2\frac{1}{2}} DR_a (1-DR_{a-1})(1-DR_a)(-\tfrac{1}{2})$$

$$+ I^{-2\frac{1}{2}} DR_{a+1} (1-DR_{a-1})(1\tfrac{1}{2}) + I^{-3\frac{1}{2}} DR_{a+1} (1-DR_{a-1})(1-DR_a)(\tfrac{1}{2}) > 0 \tag{10}$$

The first, third and fourth terms of the above inequality are positive, whereas the second term is negative. However, since death rates generally increase with age, the third term is far greater in absolute value than is the second term. Hence, (10) will be unambiguously positive. One more value for k will be tried, k=3: this trial will generate the same values that are shown in (10) plus five new combinations for (n,m) = (1,3), (2,3), (3,3), (3,1), (3,2). Since it is already known that the terms in (10) sum to an amount greater than zero, only these last five terms are shown below.

$$I^{-3\frac{1}{2}} DR_{a+2} (1-DR_{a-1})(2\tfrac{1}{2}) + I^{-4\frac{1}{2}} DR_{a+2} (1-DR_{a-1})(1-DR_a)(1\tfrac{1}{2})$$

$$+ I^{-5\frac{1}{2}} DR_{a+2} (1-DR_{a-1})(1-DR_a)(1-DR_{a+1})(\tfrac{1}{2}) +$$

$$I^{-3\frac{1}{2}} DR_a (1-DR_{a-1})(1-DR_a)(1-DR_{a+1})(-1\tfrac{1}{2}) + I^{-4\frac{1}{2}} DR_{a+1} (1-DR_{a-1})(1-DR_a)\cdot$$

$$(1-DR_{a+1})(-\tfrac{1}{2}) > 0 . \tag{11}$$

The pattern should be clear by now. The first three expressions are all positive, but the last two are negative. The first and fourth terms lend themselves to easy comparison, as well as the second and fifth terms. A comparison reveals that the first and second terms are of greater absolute value than the fourth and fifth terms, and therefore (11) is also positive. This pattern will repeat itself

for any k. The implication is that no matter what the horizon is for maintaining insurance in force, a rising discount factor will result in higher costs per unit of benefits. Moreover, as long as the discount factor rises when anticipated inflation increases ($dI/dJ > 0$), anticipated inflation will lead to higher perceived costs of life insurance by rational consumers.

APPENDIX C
CALCULATION OF THE INDICES

The index applied in adjusting life insurance values is based upon the index used in correcting the values of Obrigações Reajustáveis do Tesouro Nacional (ORTNs).[1] The manner in which the monetary correction coefficients for the ORTNs are calculated has undergone several changes, and can be viewed in six distinct stages.[2]

STAGE 1: Law 4357 of July 16, 1964, established index linkage for ORTNs. The system initially used for calculating the correction coefficients allowed for quarterly adjustments; the coefficients were based upon ratios of the average wholesale price levels that prevailed during two previous quarters. The formula for calculating the corrected values of ORTNs may be expressed as follows:

$$V_t = V_{t-1} \cdot \frac{WPI_{t-4} + WPI_{t-5} + WPI_{t-6}}{WPI_{t-7} + WPI_{t-8} + WPI_{t-9}}$$

Where

V_t is the value of the ORTN during the quarter which begins in month t,

WPI_t is the wholesale price index for month t.

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[1]Translation: Index-linked Treasury Bonds

[2]For the reader interested in greater detail, see Conjuntura Econômica, (June, 1976 issue).

STAGE 2: The ORTNs featuring quarterly adjustments were soon replaced by ORTNs featuring monthly adjustments, and the system of index calculation underwent a small modification. The formula utilized a ratio of three month moving averages of price levels, and may be expressed as follows:

$$V_t = V_{t-1} \cdot \frac{WPI_{t-4} + WPI_{t-5} + WPI_{t-6}}{WPI_{t-5} + WPI_{t-6} + WPI_{t-7}}$$

Where

V_t is the value of the ORTN for the month t, and

WPI_t is the wholesale price index for month t.

STAGE 3: The inclusion of the wholesale prices of goods produced for export in the wholesale price index led to sizable oscillations in the index, since international commodity prices are often highly volatile. These price swings generally have only indirect effects on the price levels of goods consumed domestically (e.g. although the price of coffee sold for export fluctuated wildly, the price of coffee consumed in Brazil was fixed). To avoid large movements in the indices used for monetary correction, a new "domestic supply" index was calculated. The new index, which began to be applied in July of 1967, included the wholesale prices of imported goods and of goods produced for domestic consumption, but excluded the wholesale prices of exports.

STAGE 4: A new criteria for the calculation of corrective indices was adopted in December of 1972. At this time an expectations component was introduced into the calculation of the correction. The formula was designed to give equal weights to the inflation experienced in the past and the inflation rate expected to prevail in the coming 12 months. The annual rate of expected inflation utilized in the formula was 12 percent, which amounts to a monthly compounded rate of 0.949 percent.

The new formula, which was not published at that time, had the following mathematical form:

$$V_t = 0.5 \cdot V_{t-1} \frac{WPI_{t-4} + WPI_{t-5} + WPI_{t-6}}{WPI_{t-5} + WPI_{t-6} + WPI_{t-7}} + 0.5 \cdot V_{t-1} \cdot 1.00949$$

Where

V_t is the value of the ORTN for the month t, and

WPI_t is the wholesale price index (domestic supply) for month t.

The purpose of this adjustment was to reduce the magnitude of the "feedback effect" of past price inflation upon the present. However, the underestimation of future inflation and the return to an accelerating rhythm of inflation in 1974 caused the government to abandon this method during the first quarter of 1974 and return to that previously used.

STAGE 5: In August of 1975, a new system of calculating indices was introduced. A new index was created, designed specifically for monetary correction. The index attempts to measure only the "pure inflationary tendency," independent of variations of a fortuitous or political nature (such as floods, freezes, cartels, etc.). In the long run the index closely approximates the unadjusted wholesale price index, but in terms of monthly variations it is much smoother.

STAGE 6: Finally, in May of 1976, another system was introduced. The purpose of the new system was twofold: (1) to reduce the lag between the months used in the index and the current month, and (2) to reduce the importance of the expectations component. Mathematically, the new formula may be expressed as follows:

$$V_t = 0.8 \cdot V_{t-1} \frac{WPI_{t-2} + WPI_{t-3} + WPI_{t-4}}{WPI_{t-3} + WPI_{t-4} + WPI_{t-5}} + 0.2 \cdot V_{t-1} \cdot 1.011715$$

Where

V_t is the value of the ORTN in month t, and

WPI_t is the wholesale price index (domestic supply adjusted to reflect only the pure inflationary tendency) for month t.

This formula reduced the lag between the period to be corrected and the reference period to two months. The expected rate of inflation was fixed at an annual rate of 15 percent, or a compounded monthly rate of 1.1715 percent.

APPENDIX D

DATA

TABLE 7

Probabilities

Policy Year	Lapse Rate *	Probability of Persisting given survival	Probability of Survival until end of year, male, age 35	Probability of Dying during year given survival through previous year
1	.40	.6000	.996438	.003562
2	.15	.5100	.992633	.003819
3	.10	.4590	.988562	.004101
4	.05	.4361	.984200	.004412
5	.04	.4186	.979520	.004755
6	.04	.4019	.974492	.005133
7	.03	.3899	.969085	.005549
8	.03	.3781	.963264	.006007
9	.03	.3668	.956991	.006512
10	.02	.3594	.950227	.007068
11	.02	.3522	.942928	.007681
12	.02	.3452	.935050	.008355
13	.02	.3383	.926543	.009098
14	.02	.3315	.917355	.009916
15	.02	.3249	.907432	.010817
16	.02	.3184	.896717	.011809
17	.04	.3057	.885148	.012901
18	.07	.2843	.872578	.014201
19	.18	.2331	.859118	.015425
20	1.00	.0000	.844616	.016881

* Lapse rates were obtained from Sul America Life Insurance Company. The series was truncated at twenty years, beginning in year seventeen. Mortality data was obtained from Moura (1976).

TABLE 8

Life Insurance Data*

Year	Policy Reserves	Premiums	Insurance in Force
1949		839 063	
1950	2 804	1 386 858	20 961
1951	2 752	882 997	29 111
1952	3 009	1 023 592	36 382
1953	3 285	1 191 337	46 457
1954	3 748	1 469 310	61 893
1955	4 174	1 765 447	79 169
1956	4 717	2 130 079	107 166
1957	5 332	2 595 110	141 377
1958	6 493	3 304 051	181 673
1959	6 540	3 953 885	242 048
1960	8 306	5 016 642	460 827
1961	9 981	6 071 788	550 000
1962	12 579	9 285 904	714 649
1963	16 292	13 956 777	1 154 900
1964	26 837	24 636 498	2 055 644
1965	37 827	41 908 327	3 861 391
1966	44 253	63 850 539	7 337 556
1967	50 030	96 691 700	11 021 756
1968	19 104	130 665 616	16 117 387
1969	70 973	195 089 778	23 608 307
1970	92 409	298 236 796	33 681 478
1971	107 643	393 460 178	45 807 159
1972	129 534	532 023 606	64 338 407
1973	154 735	798 941 533	84 037 209
1974	163 991	1 141 707 833	117 690 214
1975	224 433	1 658 138 984	182 138 027
1976	289 855	2 397 900 000	270 607 912
1977	364 176		400 987 266

*Values for reserves and insurance are given in thousands of new cruzeiros. Source: Instituto de Resseguros do Brasil.

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